量子光学
第三章 光场相干性及其干涉

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课程大纲

量子光学是研究光场的量子性质，及其与原子、分子等物质相互作用的物理学分支，核心内容包括光子的产生、操控与探测以及光–物质的量子相干与纠缠现象。

第一章 辐射场及其量子化 (2 次课)

第二章 **量子相干态** (3 次课)

第三章 **光场相干性及其干涉** (3 次课)

第四章 **光场与原子相互作用** (4 次课)

第五章 热库系统与主方程 (2 次课)

第六章 激光冷却技术 (2 次课)

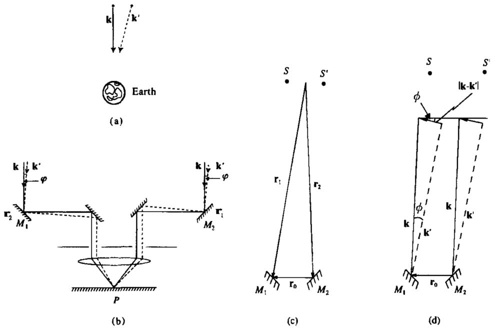
第七章 光学腔和光力耦合系统 (2 次课)

参考资料

- 《量子光学》，郭光灿，周祥发，科学出版社
- Quantum Optics, Marlan O. Scully, M. Suhail Zubairy, Cambridge University Press
- Lecture of Professor Farhan Rana
- 维基百科 (wikipedia.org)

第三章

迈克尔逊恒星干涉仪 (Michelson stellar interferometer)



$$I = \kappa \langle \boldsymbol{E}^* \boldsymbol{E} \rangle = \kappa \left\langle \left| E_k (e^{i\boldsymbol{k}\cdot\boldsymbol{r}_1} + e^{i\boldsymbol{k}\cdot\boldsymbol{r}_2}) + E_{k'} (e^{i\boldsymbol{k}'\cdot\boldsymbol{r}_1} + e^{i\boldsymbol{k}'\cdot\boldsymbol{r}_2}) \right|^2 \right\rangle$$

$$= 2\kappa \left\langle |E_k|^2 + |E_{k'}|^2 + |E_k|^2 \cos \boldsymbol{k}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2) + |E_{k'}|^2 \cos \boldsymbol{k}'\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2) \right\rangle$$

- 信号经过过滤后，光场频率相同
- 考虑恒星的热辐射 $\langle \boldsymbol{E}_k \rangle = \langle \boldsymbol{E}_{k'} \rangle = 0$, $\langle \boldsymbol{E}_k^* \boldsymbol{E}_{k'} \rangle = 0$
- 设 $\langle |\boldsymbol{E}_k| \rangle = \left\langle |\boldsymbol{E}_{k'}|^2 \right\rangle = I_0$

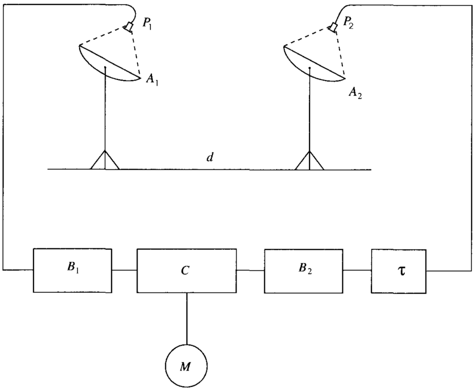
迈克尔逊恒星干涉仪

$$
\begin{aligned}
I &= 2\kappa I_0 \left[2 + \cos\left(\boldsymbol{k}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)\right) + \cos\left(\boldsymbol{k}'\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)\right)\right] \\
&= 4\kappa I_0 \left[1 + \cos\frac{(\boldsymbol{k}+\boldsymbol{k}')\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)}{2}\cos\frac{(\boldsymbol{k}-\boldsymbol{k}')\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)}{2}\right] \\
&= 4\kappa I_0 \left[1 + \cos\frac{(\boldsymbol{k}+\boldsymbol{k}')\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)}{2}\cos\frac{\pi r_0 \phi}{\lambda}\right]
\end{aligned}
$$

- 近似 $(\boldsymbol{k}-\boldsymbol{k}')\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2) \approx \phi k r_0$, and $k = 2\pi/\lambda$
- 这个方法已经应用到了多个双星系统
- 大气层的影响和仪器的涨落使得 $\cos\frac{(\boldsymbol{k}+\boldsymbol{k}')\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)}{2}$ 变化较大，限制了这个方法的精度

Hanbury-Brown-Twiss interferometer

Fig. 4.6
Schematic diagram of the Hanbury Brown-Twiss stellar intensity interferometer. Here P_1 and P_2 are the photodetectors, A_1 and A_2 are the mirrors, B_1 and B_2 are the amplifiers, τ is the delay time, C is a multiplier, and M is the integrator.



Hanbury-Brown-Twiss interferometer

在两个不同位置 $\boldsymbol{r}_1$ 和 $\boldsymbol{r}_2$ 测量光场强度:

$$I(\boldsymbol{r}_i,t)=\kappa\left[|E_k|^2+|E_{k'}|^2+E_kE_{k'}e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot r_i}+E^*_{k'}E_ke^{-i(\boldsymbol{k}-\boldsymbol{k}')\cdot r_i}\right]$$

两个探测器的符合计数

$$\begin{aligned}\langle I(\boldsymbol{r}_1,t)I(\boldsymbol{r}_2,t)\rangle =&\kappa^2\left\langle\left[|E_k|^2+|E_{k'}|^2+E_kE_{k'}e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot r_1}+E^*_{k'}E_ke^{-i(\boldsymbol{k}-\boldsymbol{k}')\cdot r_1}\right]\right.\\ &\left.\left[|E_k|^2+|E_{k'}|^2+E_kE_{k'}e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot r_2}+E^*_{k'}E_ke^{-i(\boldsymbol{k}-\boldsymbol{k}')\cdot r_2}\right]\right\rangle\\ =&\kappa^2\left\langle(|E_k|^2+|E_{k'}|^2)^2\right\rangle+\left\langle|E_k|^2\right\rangle\left\langle|E_{k'}|^2\right\rangle 2\cos\left[(\boldsymbol{k}-\boldsymbol{k}')\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)\right]\end{aligned}$$

- **同样假定热辐射场**
- **没有** $\cos[(\boldsymbol{k}+\boldsymbol{k}')\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)/2]\leftarrow$ **高频**

相关函数

在量子光学中，关联函数 (Correlation function) 可以描述电磁辐射场的统计性质和相干性

- Coherence functions, Roy Glauber and others in 1960s
- 相干波具有确定的相位关系
- 杨氏双缝实验
- 干涉仪：Mach-Zehnder 干涉仪
- Hanbury Brown-Twiss 实验

量子相关函数

电场算符写成正频和负频部分

$$\hat{E}(\boldsymbol{r},t) = \hat{E}^{(+)}(\boldsymbol{r},t) + \hat{E}^{(-)}(\boldsymbol{r},t)$$

$$\hat{E}^{(+)} = i\sum_{\boldsymbol{k},\sigma}\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\hat{a}_{\boldsymbol{k},\sigma}e^{-i\omega_k t + i\boldsymbol{k}\cdot\boldsymbol{r}}, \quad \hat{E}^{(-)} = \left[\hat{E}^{(+)}(\boldsymbol{r},t)\right]^\dagger$$

- 光电探测过程中，原子吸收一个光子同时会发射一个电子
- 电信号放大后被读取

量子相关函数

电场算符写成正频和负频部分

$$\hat{E}(\boldsymbol{r},t) = \hat{E}^{(+)}(\boldsymbol{r},t) + \hat{E}^{(-)}(\boldsymbol{r},t)$$

$$\hat{E}^{(+)} = i\sum_{\boldsymbol{k},\sigma}\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\hat{a}_{\boldsymbol{k},\sigma}e^{-i\omega_k t+i\boldsymbol{k}\cdot\boldsymbol{r}}, \quad \hat{E}^{(-)} = \left[\hat{E}^{(+)}(\boldsymbol{r},t)\right]^\dagger$$

- 光的吸收过程中，光子湮灭，只有正频部分起作用
- 量子探测中，正频和负频不对称：实际探测的是 $\hat{E}^{(+)}$
- 经典探测：探测器不仅会吸收电磁量子，也会发射电磁量子，$\hat{E}^{(+)}$ 和 $\hat{E}^{(-)}$ 都起作用

量子相关函数

- 电磁场在时空点 $(\boldsymbol{r}, t)$ 处被吸收一个光子，场的状态从 $|i\rangle$ 跃迁到 $|f\rangle$

$$\langle f|\hat{E}^{(+)}|i\rangle$$

- 一般无法确定终态，为得到总跃迁概率，需要对所有末态求和

$$\begin{aligned} w_1(\boldsymbol{r}, t) &= \sum_f \left| \langle f|\hat{E}^{(+)}(\boldsymbol{r}, t)|i\rangle \right|^2 \\ &= \sum_f \langle i|\hat{E}^{(-)}(\boldsymbol{r}, t)|f\rangle \langle f|\hat{E}^{(+)}|i\rangle \\ &= \langle i|\hat{E}^{(-)}(\boldsymbol{r}, t)\hat{E}^{(+)}(\boldsymbol{r}, t)|i\rangle \end{aligned}$$

- 用密度矩阵替代

$$w_1(\boldsymbol{r}, t) = \mathrm{Tr}\left[\rho \hat{E}^{(-)}(\boldsymbol{r}, t)\hat{E}^{(+)}(\boldsymbol{r}, t)\right]$$

量子相关函数

定义光场在不同时空点的一阶相关函数

$$G^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2;t_1,t_2)=\mathrm{Tr}\left[\rho\hat{E}^{(-)}(\boldsymbol{r}_1,t_1)\hat{E}^{(+)}(\boldsymbol{r}_2,t_2)\right]$$

利用时间平移不变性，设 $\tau=t_2-t_1$

$$G^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2;t_1,t_2)=G^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2,\tau)$$

定义两个时空点之间的相干性

量子相关函数

两个探测器在时空点 $(\boldsymbol{r}_1, t_1)$ 和 $(\boldsymbol{r}_2, t_2)$ 处探测到光子的联合概率

$$w_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t_1, t_2) = \left| \langle f | \hat{E}^{(+)}(\boldsymbol{r}_2, t_2) \hat{E}^{(+)}(\boldsymbol{r}_1, t_1) | i \rangle \right|^2$$

同样，对所有的末态求和

$$w_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t_1, t_2) = \mathrm{Tr}\left[\rho \hat{E}^{(-)}(\boldsymbol{r}_1, t_1) \hat{E}^{(-)}(\boldsymbol{r}_2, t_2) \hat{E}^{(+)}(\boldsymbol{r}_2, t_2) \hat{E}^{(+)}(\boldsymbol{r}_1, t_1)\right]$$

$$\downarrow$$

二阶相干函数

量子相关函数

- 注意算符的排序：产生算子总排在湮灭算子的左边

正规排序

- 归一化的一阶相干度 ($X_i = (\boldsymbol{r}_i, t_i)$)

$$g^{(1)}(X_1, X_2) = \frac{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(+)}(X_2) \right\rangle}{\sqrt{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(+)}(X_1) \right\rangle \left\langle \hat{E}^{(-)}(X_2)\hat{E}^{(+)}(X_2) \right\rangle}}$$

- 归一化的二阶相干度

$$g^{(2)}(X_1, X_2) = \frac{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(-)}(X_2)\hat{E}^{(+)}(X_2)\hat{E}^{(+)}(X_1) \right\rangle}{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(+)}(X_1) \right\rangle \left\langle \hat{E}^{(-)}(X_2)\hat{E}^{(+)}(X_2) \right\rangle}$$

量子相关函数

类似，可以定义 n 阶相干函数

$$
\begin{aligned}
&G^{(n)}(\boldsymbol{r}_1,\dots,\boldsymbol{r}_{2n};t_1,\dots,t_{2n})\\
&=\mathrm{Tr}\Big[\rho e^{(-)}(\boldsymbol{r}_1,t_1)\dots E^{(-)}(\boldsymbol{r}_n,t_n)E^{(+)}(\boldsymbol{r}_{n+1},t_{n+1})\dots E^{(+)}(\boldsymbol{r}_{2n},t_{2n})\Big]
\end{aligned}
$$

量子相关函数

对于单模平面波

$$\hat{E}^{(+)}(\boldsymbol{r},t) = i\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}} e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\hat{a}$$

在空间点 $\boldsymbol{r}_i = \boldsymbol{r}$，相干度可以表示为

$$g^{(1)}(\tau) = \frac{\left\langle \hat{a}^\dagger(t+\tau)\hat{a}(t)\right\rangle}{\langle \hat{a}^\dagger\hat{a}\rangle}$$

$$g^{(2)}(\tau) = \frac{\left\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\right\rangle}{\langle \hat{a}^\dagger\hat{a}\rangle^2}$$

有不等式

$$\left|\left\langle \hat{a}^\dagger(t+\tau)\hat{a}(t)\right\rangle\right|^2 \leq \left\langle \hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\right\rangle \left\langle \hat{a}^\dagger(t)\hat{a}(t)\right\rangle$$

$$\rightarrow \left|g^{(1)}(\tau)\right| \leq 1$$

量子相关函数

- 若 $\left|g^{(1)}(X_1, X_2)\right| = 1$，则称光场在时空点 X_1 和 X_2 具有一阶相干性
- 对于整个光场，所有的时空点都满足 $\left|g^{(1)}(X_1, X_2)\right| = 1$ 的场，称为具有一阶相干性
- 相干性 $\rightarrow$ 反映光场的噪声强弱
- 对任何时空点，满足

$$\left|g^{(m)}(X_1, X_2, \ldots, X_{2m})\right| = 1, \quad 1 \leq m \leq n$$

称为 n 阶相干光场

量子相干态

对于相干态 $|\alpha\rangle$:

$$g^{(1)}(\tau) = \frac{\langle \hat{a}^\dagger e^{i\omega\tau} \hat{a} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle} = e^{i\omega\tau}$$

n 阶相干度

$$\left| g^{(n)}(\tau) \right| = 1$$

量子相关函数

- **多模热光场**

$$\rho_k = \sum_{n_k=0} \frac{\bar{n}_k^{n_k}}{(1+\bar{n}_k)^{1+n_k}} |n_k\rangle\langle n_k|$$

其中 $\bar{n}_k = [\exp(\hbar\omega_k/k_B T) - 1]^{-1}$

- **可以得到**

$$g^{(1)}(\tau) = \frac{\sum_k \bar{n}_k \omega_k \exp(-i\omega_k \tau)}{\sum_k \bar{n}_k \omega_k}$$

其中 $\tau = t_2 - t_1 - (z_2 - z_1)/c$

量子相关函数

均匀加宽的热光场，满足洛伦兹线性

$$\bar{n}_k \omega_k \sim \frac{\gamma/\pi}{(\omega_k - \omega_0)^2 + \gamma^2}$$

把求和换成积分

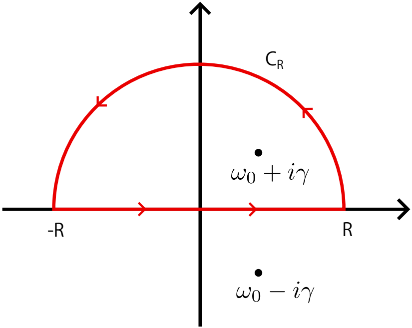
$$\sum_k \to \int_0^\infty \frac{L}{\pi}\, \mathrm{d}k = \int_0^\infty \frac{L}{\pi c}\, \mathrm{d}\omega_k$$

洛伦兹谱分布热光场的一阶相干度

$$g^{(1)}(\tau) = \int_0^\infty \frac{\gamma/\pi \exp(-i\omega_k \tau)}{(\omega_k - \omega_0)^2 + \gamma^2}\, \mathrm{d}\omega_k$$

量子相关函数

设 $f(z)=\frac{\gamma/\pi}{(z-\omega_0)^2+\gamma^2}e^{-iz\tau}$



$$\int_{-R}^{R} f(z)\,\mathrm{d}z+\int_{C_R} f(z)\,\mathrm{d}z=2\pi i\,\mathrm{Res}[f(z),\omega_0+i\gamma]$$

留数定理

量子相关函数

$$f(z) = \frac{1}{2\pi i}\left(\frac{1}{z-\omega_0-i\gamma} - \frac{1}{z-\omega_0+i\gamma}\right)e^{-iz\tau}$$

得到

$$\mathrm{Res}[f(z),\omega_0+i\gamma] = \frac{1}{2\pi i}e^{-i(\omega_0+i\gamma)\tau}$$
$$\mathrm{Res}[f(z),\omega_0-i\gamma] = -\frac{1}{2\pi i}e^{-i(\omega_0-i\gamma)\tau}$$

- 如果 $\tau < 0$，环路积分 $\int_{C_R} f(z)\,\mathrm{d}z = 0$，包含极点 $\omega_0 + i\gamma$

$$\int_{-R}^{R} f(z)\,\mathrm{d}z = e^{-i\omega\tau+\gamma\tau}$$

- 如果 $\tau > 0$，则 $\int_{C_R} f(z)\,\mathrm{d}z$ 不等于 0，环路该取下半环路

$$\int_{-R}^{R} f(z)\,\mathrm{d}z = e^{-i\omega\tau-\gamma\tau}$$

量子相关函数

$$f(z) = \frac{1}{2\pi i}\left(\frac{1}{z-\omega_0-i\gamma} - \frac{1}{z-\omega_0+i\gamma}\right)e^{-iz\tau}$$

得到

$$\mathrm{Res}[f(z),\omega_0+i\gamma] = \frac{1}{2\pi i}e^{-i(\omega_0+i\gamma)\tau}$$

$$\mathrm{Res}[f(z),\omega_0-i\gamma] = -\frac{1}{2\pi i}e^{-i(\omega_0-i\gamma)\tau}$$

最后得到

$$g^{(1)}(\tau) = e^{-i\omega\tau-\gamma|\tau|}$$

量子相关函数

光束的谱函数是高斯线性函数

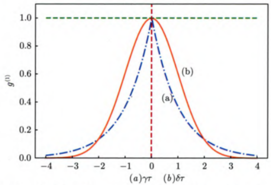
$$\bar{n}_k \omega_k \sim \frac{1}{\sqrt{2\pi\delta^2}} \exp\left[-(\omega_k - \omega_0)^2 / 2\delta^2\right]$$

一阶相干函数

$$g^{(1)}(\tau) \sim \frac{1}{\sqrt{2\pi\delta^2}} \exp\left[-\frac{(\omega_k - \omega_0)^2}{2\delta^2}\right] e^{-i\omega_k \tau} \, \mathrm{d}\omega_k$$

经过积分后，得到

$$g^{(1)}(\tau) = e^{-i\omega_0 \tau - \frac{1}{2}\delta^2 \tau^2}$$



经典热光场的一阶相干度，其中 (a) 对应洛伦兹线型热光场，(b) 对应高斯线型热光场

量子相关函数

单模热光场的二阶相干度

$$g^{(2)}(\tau)=\frac{\langle\hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\rangle}{\langle\hat{a}^\dagger\hat{a}\rangle^2}=\frac{\left\langle(\hat{a}^\dagger)^2\hat{a}^2\right\rangle}{\langle\hat{a}^\dagger\hat{a}\rangle^2}$$

- 单模下，二阶相干度与时间没有关系
- 利用对易关系

$$g^{(2)}=\frac{\langle\hat{n}(\hat{n}-1)\rangle}{\langle\hat{n}\rangle^2}=\frac{\left\langle\hat{n}^2\right\rangle-\langle\hat{n}\rangle}{\langle\hat{n}\rangle^2}=2$$

对 n 个光子的测量将使光子数减少一个，因此，第二次测量只能发现 $n-1$ 个光子

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量子相关函数

多模热光场的二阶相干度

$$g^{(2)}(X_1, X_2) = \frac{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(-)}(X_2)\hat{E}^{(+)}(X_2)\hat{E}^{(+)}(X_1) \right\rangle}{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(+)}(X_1) \right\rangle \left\langle \hat{E}^{(-)}(X_2)\hat{E}^{(+)}(X_2) \right\rangle}$$

- **电场**

$$\hat{E}^{(+)}(\boldsymbol{r}, t) = i \sum_{\boldsymbol{k},\sigma} \hat{e}_\sigma \sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}} \hat{a}_{\boldsymbol{k},\sigma} e^{-i\omega_k t + i\boldsymbol{k}\cdot\boldsymbol{r}}$$

量子相关函数

Wick's theorem

$$\hat{A}\hat{B} =: \hat{A}\hat{B} : + \hat{A}^{\bullet}\hat{B}^{\bullet}$$

- $:\hat{A}\hat{B}:$ 表示算符的正规排序
- $\hat{A}^{\bullet}\hat{B}^{\bullet}$ 表示算符的收缩

$$\hat{A}\hat{B}\hat{C}\cdots =: \hat{A}\hat{B}\hat{C}\cdots : + \sum_{\text{singles}} :\hat{A}^{\bullet}\hat{B}^{\bullet}\hat{C}\cdots : + \sum_{\text{doubles}} \cdots + \ldots$$

量子相关函数

$$g^{(2)}(X_1,X_2)=\frac{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(-)}(X_2)\hat{E}^{(+)}(X_2)\hat{E}^{(+)}(X_1)\right\rangle}{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(+)}(X_1)\right\rangle\left\langle \hat{E}^{(-)}(X_2)\hat{E}^{(+)}(X_2)\right\rangle}$$

设 $\hat{E}^{(+)}=\sum_k A\sqrt{\omega_k}\hat{a}_k e^{-i\omega_{\boldsymbol{k}}t}$

- **分子**

$$\begin{aligned}
&\sum_{k,l,m,n}\sqrt{\omega_k\omega_l\omega_m\omega_n}\left\langle \hat{a}_k^\dagger\hat{a}_l^\dagger\hat{a}_m\hat{a}_n\right\rangle e^{i(\omega_k-\omega_n)t}e^{i(\omega_l-\omega_m)(t+\tau)}\\
=&\sum_{k,l,m,n}\sqrt{\omega_k\omega_l\omega_m\omega_n}\left(\left\langle \hat{a}_k^\dagger\hat{a}_m\right\rangle\left\langle \hat{a}_l^\dagger\hat{a}_n\right\rangle+\left\langle \hat{a}_k^\dagger\hat{a}_n\right\rangle\left\langle \hat{a}_l^\dagger\hat{a}_m\right\rangle\right)\\
&\times e^{i(\omega_k-\omega_n)t}e^{i(\omega_l-\omega_m)(t+\tau)}\\
=&\sum_{k,l}\left(\omega_k\omega_l\left\langle \hat{a}_k^\dagger\hat{a}_k\right\rangle\left\langle \hat{a}_l^\dagger\hat{a}_l\right\rangle e^{i(\omega_l-\omega_k)\tau}+\omega_k\omega_l\left\langle \hat{a}_k^\dagger\hat{a}_k\right\rangle\left\langle \hat{a}_l^\dagger\hat{a}_l\right\rangle\right)
\end{aligned}$$

- **分母**

$$\sum_{k,l}\omega_k\omega_l\left\langle \hat{a}_k^\dagger\hat{a}_k\right\rangle\left\langle \hat{a}_l^\dagger\hat{a}_l\right\rangle$$

量子相关函数

$$g^{(2)}(X_1,X_2)=\frac{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(-)}(X_2)\hat{E}^{(+)}(X_2)\hat{E}^{(+)}(X_1)\right\rangle}{\left\langle \hat{E}^{(-)}(X_1)\hat{E}^{(+)}(X_1)\right\rangle\left\langle \hat{E}^{(-)}(X_2)\hat{E}^{(+)}(X_2)\right\rangle}$$

设 $\hat{E}^{(+)}=\sum_k A\sqrt{\omega_k}\hat{a}_k e^{-i\omega_{\boldsymbol{k}}t}$

得到

$$\begin{aligned}g^{(2)}(\tau)&=\frac{\sum_{k,l}\left(\omega_k\omega_l\left\langle\hat{a}_k^\dagger\hat{a}_k\right\rangle\left\langle\hat{a}_l^\dagger\hat{a}_l\right\rangle e^{i(\omega_l-\omega_k)\tau}+\omega_k\omega_l\left\langle\hat{a}_k^\dagger\hat{a}_k\right\rangle\left\langle\hat{a}_l^\dagger\hat{a}_l\right\rangle\right)}{\sum_{k,l}\omega_k\omega_l\left\langle\hat{a}_k^\dagger\hat{a}_k\right\rangle\left\langle\hat{a}_l^\dagger\hat{a}_l\right\rangle}\\&=1+\frac{G^{(1)}(\tau)G^{(1)*}(\tau)}{\left\langle\hat{I}\right\rangle^2}\\&=1+\left|g^{(1)}(\tau)\right|^2\end{aligned}$$

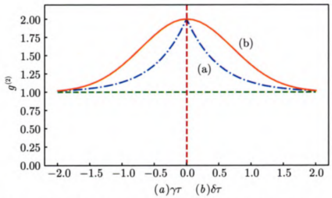
量子相关函数

- 洛伦兹线型

$$g^{(2)} = 1 + e^{-2\gamma|\tau|}$$

- 高斯线型

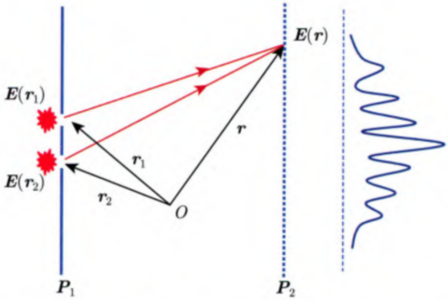
$$g^{(2)}(\tau) = 1 + e^{-\delta^2\tau^2}$$



经典热光场的二阶相干度，其中 (a) 对应洛伦兹线型热光场，(b) 对应高斯线型热光场

- 若 τ 很小，热光场可以认为是一阶相干光场
- 但不是二阶相干光场

量子相关函数



$$\hat{E}^{(+)}(\boldsymbol{r}, t) = u_1 \hat{E}^{(+)}(\boldsymbol{r}_1, t_1) + u_2 \hat{E}^{(+)}(\boldsymbol{r}_2, t_2)$$

量子相关函数

探测器平均光强

$$
\begin{aligned}
\langle I(\boldsymbol{r},t)\rangle =& \left\langle \hat{E}^{(-)}(\boldsymbol{r},t)\hat{E}^{(+)}(\boldsymbol{r},t)\right\rangle \\
=& |u_1|^2 \left\langle \hat{E}^{(-)}(\boldsymbol{r}_1,t_1)\hat{E}^{(+)}(\boldsymbol{r}_1,t_1)\right\rangle + |u_1|^2 \left\langle \hat{E}^{(-)}(\boldsymbol{r}_1,t_1)\hat{E}^{(+)}(\boldsymbol{r}_1,t_1)\right\rangle \\
&+ 2\,\mathrm{Re}\left[u_1^* u_2 \left\langle \hat{E}^{(-)}(\boldsymbol{r}_1,t_1)\hat{E}^{(+)}(\boldsymbol{r}_2,t_2)\right\rangle\right] \\
=& \langle I_1(\boldsymbol{r},t)\rangle + \langle I_2(\boldsymbol{r},t)\rangle \\
&+ 2\sqrt{\langle I_1(\boldsymbol{r},t)\rangle \langle I_2(\boldsymbol{r},t)\rangle}\, g^{(1)}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2)\cos[k(s_1-s_2)+\phi]
\end{aligned}
$$

- **条纹可见度**

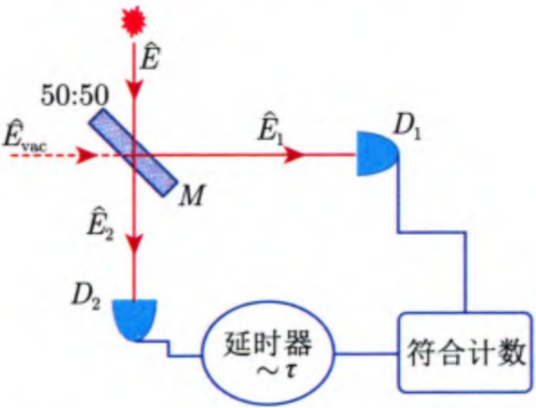
$$
\begin{aligned}
\mathcal{V} &= \frac{\langle I(\boldsymbol{r},t)\rangle_{\max} - \langle I(\boldsymbol{r},t)\rangle_{\min}}{\langle I(\boldsymbol{r},t)\rangle_{\max} + \langle I(\boldsymbol{r},t)\rangle_{\min}} \\
&= \frac{2\sqrt{\langle I_1(\boldsymbol{r},t)\rangle \langle I_2(\boldsymbol{r},t)\rangle}}{\langle I(\boldsymbol{r},t)\rangle_{\max} + \langle I(\boldsymbol{r},t)\rangle_{\min}} g^{(1)}(\boldsymbol{r}_1,t_1,\boldsymbol{r}_2,t_2)
\end{aligned}
$$

- **一阶相干度直接对应干涉条纹的可见度**

量子相关函数

- 传统光学研究的实质都是光场的一阶相干性
- 直到 1956 年，Hanbury Brown 和 Twiss 开创了一类新型的光学干涉实验 $\rightarrow$ 首次观测光场的二阶相干性
- 量子光学的奠基性实验

量子相关函数



量子相关函数

经典处理

- 假设入射光强为 $I(t)$ 且 D_1 和 D_2 接收到的瞬时强度相等

$$I_1(t) = I_2(t) = \frac{1}{2}I(t)$$

- 二阶相干度在经典下为: $g_c^{(2)}(\tau) = \frac{\langle I_1(t)I_2(t+\tau)\rangle}{\langle I_1(t)\rangle\langle I_2(t)\rangle}$
- 利用柯西不等式，得到

$$\langle I_1(t)I_2(t)\rangle \geq \langle I_1(t)\rangle \langle I_2(t)\rangle$$

- 不等式

$$[I(t_1)I(t_1+\tau)+\cdots+I(t_N)I(t_N+\tau)]^2 \leq$$
$$[I^2(t_1)+\cdots+I^2(t_N)][I^2(t_1+\tau)+\cdots+I^2(t_N+\tau)]$$

得到

$$\langle I_1(t)I_2(t)\rangle \geq \langle I_1(t)I_2(t+\tau)\rangle$$

量子相关函数

经典处理

- 假设入射光强为 $I(t)$ 且 D_1 和 D_2 接收到的瞬时强度相等

$$I_1(t) = I_2(t) = \frac{1}{2} I(t)$$

- 二阶相干度在经典下为: $g_c^{(2)}(\tau) = \frac{\langle I_1(t) I_2(t+\tau) \rangle}{\langle I_1(t) \rangle \langle I_2(t) \rangle}$
- 满足

$$g_c^{(2)}(0) \geq 1, \quad g_c^{(2)}(0) \geq g_c^{(2)}(\tau)$$

量子相关函数

量子处理

$$\hat{E}_1 = \frac{\hat{E} + i\hat{E}_{\text{vac}}}{\sqrt{2}}, \quad \hat{E}_2 = \frac{\hat{E}_{\text{vac}} + i\hat{E}}{\sqrt{2}}$$

- 相干光场: $g^{(2)}(\tau) = 1$
- 热光场: $g^{(2)}(\tau) = 1 + g^{(1)}(\tau) \geq 1$
- 单模热光场: $g^{(2)}(\tau) = 2$ 与 τ 无关
- 当入射光场是 Fock 态？

量子相关函数

$$g^{(2)} = \frac{\left\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\right\rangle}{\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle \langle \hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\rangle}$$

- 当入射光场是 Fock 态

$$g^{(2)}(\tau) = g^{(2)}(0) = \begin{cases} 1 - \frac{1}{n}, & n \geq 2 \\ 0, & n = 0, 1 \end{cases}$$

- 单模光场

$$\begin{aligned} g^{(2)}(\tau) = g^{(0)} &= \frac{\left\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a}\right\rangle}{\langle \hat{n}\rangle^2} \\ &= \frac{\left\langle \hat{a}^\dagger (\hat{a}\hat{a}^\dagger - 1)\hat{a}\right\rangle}{\langle \hat{n}\rangle^2} = 1 + \frac{\left\langle \Delta\hat{n}^2\right\rangle - \langle \hat{n}\rangle}{\langle \hat{n}\rangle^2} \\ &= 1 + \frac{\mathcal{M}}{\langle n\rangle} \end{aligned}$$

光场的非经典效应与光子数的 Mandel 参数的关系

量子相关函数

奇偶相干态

$$|\alpha\rangle_o = \mathcal{N}_o(|\alpha\rangle - |-\alpha\rangle), \quad |\alpha\rangle_e = \mathcal{N}_e(|\alpha\rangle + |-\alpha\rangle),$$
$$\hat{a}\,|\alpha\rangle_o = \alpha\coth^{1/2}|\alpha|^2\,|\alpha\rangle_e\,, \quad \hat{a}\,|\alpha\rangle_e = \alpha\tanh^{1/2}|\alpha|^2\,|\alpha\rangle_o$$

粒子数

$${}_o\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle_o = |\alpha|^2\coth|\alpha|^2, \qquad {}_e\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle_e = |\alpha|^2\tanh|\alpha|^2$$

二阶相干函数

$$g_o^{(2)}(0) = \frac{{}_o\left\langle\hat{a}^{\dagger 2}\hat{a}^2\right\rangle_o}{{}_o\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle_o^2}$$

量子相关函数

奇偶相干态

$$|\alpha\rangle_o = \mathcal{N}_o(|\alpha\rangle - |-\alpha\rangle), \quad |\alpha\rangle_e = \mathcal{N}_e(|\alpha\rangle + |-\alpha\rangle),$$
$$\hat{a}\,|\alpha\rangle_o = \alpha \coth^{1/2}|\alpha|^2\,|\alpha\rangle_e\,, \quad \hat{a}\,|\alpha\rangle_e = \alpha \tanh^{1/2}|\alpha|^2\,|\alpha\rangle_o$$

粒子数

$${}_o\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle_o = |\alpha|^2 \coth|\alpha|^2, \qquad {}_e\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle_e = |\alpha|^2 \tanh|\alpha|^2$$

二阶相干函数

$$g_o^{(2)}(0) = \tanh^2|\alpha|^2 = \left(\frac{e^{|\alpha|^2} - e^{-|\alpha|^2}}{e^{|\alpha|^2} + e^{-|\alpha|^2}}\right)^2 < 1$$
$$g_e^{(2)}(0) = \coth^2|\alpha|^2 = \left(\frac{e^{|\alpha|^2} + e^{-|\alpha|^2}}{e^{|\alpha|^2} - e^{-|\alpha|^2}}\right)^2 > 1$$

量子相关函数

奇偶相干态

$$|\alpha\rangle_o = \mathcal{N}_o(|\alpha\rangle - |-\alpha\rangle), \quad |\alpha\rangle_e = \mathcal{N}_e(|\alpha\rangle + |-\alpha\rangle),$$
$$\hat{a}\,|\alpha\rangle_o = \alpha \coth^{1/2}|\alpha|^2\,|\alpha\rangle_e\,, \quad \hat{a}\,|\alpha\rangle_e = \alpha \tanh^{1/2}|\alpha|^2\,|\alpha\rangle_o$$

粒子数

$$_o\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle_o = |\alpha|^2\coth|\alpha|^2, \qquad _e\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle_e = |\alpha|^2\tanh|\alpha|^2$$

二阶相干函数

- 奇相干态光子数呈现非经典效应
- 偶相干态中光子数呈现超泊松分布

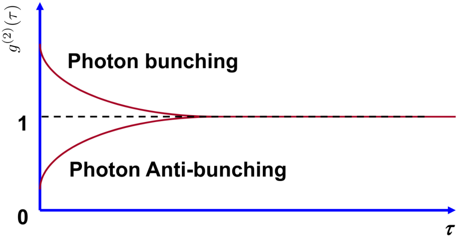
量子相关函数

- 群聚效应 (photon bunching)

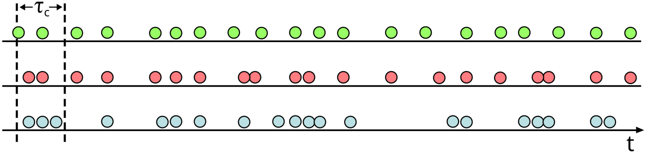
$$g^{(2)}(\tau) < g^{(2)}(0)$$

- 反群聚效应 (photon anti-bunching)

$$g^{(2)}(\tau) > g^{(2)}(0)$$



量子相关函数



Photon detections as function of time for a) antibunched, b) random, and c) bunched light

单模光场

$$g^{(2)}(\tau) = g^{(2)}(0) = 1 + \frac{\left\langle \Delta\hat{n}^2 \right\rangle - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2}$$

量子相关函数

多模光场

$$g^{(2)}(\tau) = 1 + \frac{\int P(|\alpha_j|)\left(\sum_j |\alpha_j|^2 - \left\langle \hat{a}_j^\dagger \hat{a}_j \right\rangle\right)^2 \mathrm{d}^2\{\alpha_j\}}{(\sum_j \left\langle \hat{a}_j^\dagger \hat{a}_j \right\rangle)^2}$$

- $P(|\alpha_j|)$ 为光场的 P-表示函数

$$\left\langle \hat{a}_j^\dagger \hat{a}_j \right\rangle = \int P(|\alpha_j|)\alpha_j^* \alpha_j \mathrm{d}^2\{\alpha_j\}$$

- 对于经典光场 $P(|\alpha_j|) \geq 0$，所以 $g^{(2)}(0) \geq 0$，并且 $g^{(2)}(\tau) < g^{(2)}(0)$

  经典光场呈现光子的群聚效应

- 对于量子光场，$P(|\alpha_j|)$ 不一定是正定

量子相关函数

多模光场

$$g^{(2)}(\tau)=1+\frac{\int P(|\alpha_j|)\left(\sum_j|\alpha_j|^2-\left\langle\hat{a}_j^\dagger\hat{a}_j\right\rangle\right)^2\mathrm{d}^2\{\alpha_j\}}{(\sum_j\left\langle\hat{a}_j^\dagger\hat{a}_j\right\rangle)^2}$$

- 亚泊松分布 $\rightarrow$ 反群聚效应
- 反群聚效应 $\overset{?}{\rightarrow}$ 亚泊松分布

光子数统计和群聚效应刻画的是光场两种不同特性

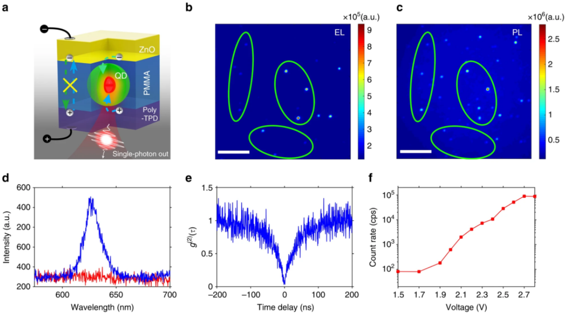
量子相关函数

多模光场

$$g^{(2)}(\tau) = 1 + \frac{\int P(|\alpha_j|)\left(\sum_j |\alpha_j|^2 - \left\langle \hat{a}_j^\dagger \hat{a}_j \right\rangle\right)^2 \mathrm{d}^2\{\alpha_j\}}{(\sum_j \left\langle \hat{a}_j^\dagger \hat{a}_j \right\rangle)^2}$$

- 不断减弱光强是否可以使得光场呈现反群聚效应
- 光场通过非线性过程除去成群的光子
- 控制发光源的发光时间间隔

量子相关函数



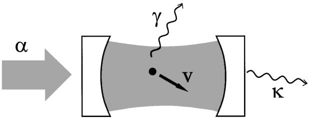
Optical properties of a single-photon device. **a** Schematic diagram of the key components of the device. **b**, Electroluminescence (driven at 2.4 V) and **c**, the corresponding photoluminescence (excited by 450 nm continuous wave laser) microscopic images. Scale bar: 5 μm. Green circles in **b** and **c** are shown as visual guides to highlight the relevant dots. **d** Electroluminescence spectrum (blue curve, driven at 2.8 V). The noise counts from the detector are also plotted (red curve). **e** $g^{(2)}(\tau)$ curve of a quantum-dot driven at 2.6 V. **f** Averaged electroluminescence counts of a single dot under different driving voltages (y-axis in log-scale)

Lin et al., Nat. Commun. 8, 1132 (2017)

- Room temperature: $g^{(2)}(0) < 0.05$

量子相关函数

一个二能级原子与光学腔耦合

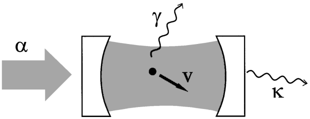


$$
\begin{aligned}
H = & \varepsilon_e |e\rangle\langle e| + \varepsilon_g |g\rangle\langle g| + \hbar\omega_c \hat{a}^\dagger \hat{a} \\
& + g(\hat{a} |e\rangle\langle g| + \hat{a}^\dagger |g\rangle\langle e|) \\
& + i\alpha(\hat{a} e^{i\omega_c t} - \hat{a}^\dagger e^{-i\omega_c t})
\end{aligned}
$$

- 激光作为激发源泵浦光学腔
- 光学腔使原子从基态 $|g\rangle$ 跳到激发态
- 处在激发态的原子耗散，回到基态，产生一个光子

量子相关函数

一个二能级原子与光学腔耦合

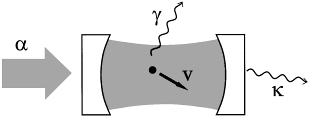


$$
\begin{aligned}
H =& \varepsilon_e |e\rangle\langle e| + \varepsilon_g |g\rangle\langle g| + \hbar\omega_c \hat{a}^\dagger \hat{a} \\
&+ g(\hat{a}|e\rangle\langle g| + \hat{a}^\dagger |g\rangle\langle e|) \\
&+ i\alpha(\hat{a}e^{i\omega_c t} - \hat{a}^\dagger e^{-i\omega_c t})
\end{aligned}
$$

- 空间维度：$2N$

量子相关函数

一个二能级原子与光学腔耦合



$$
\begin{aligned}
H =&\varepsilon_e |e\rangle\langle e| + \varepsilon_g |g\rangle\langle g| + \hbar\omega_c \hat{a}^\dagger \hat{a} \\
&+ g(\hat{a}|e\rangle\langle g| + \hat{a}^\dagger |g\rangle\langle e|) \\
&+ i\alpha(\hat{a}e^{i\omega_c t} - \hat{a}^\dagger e^{-i\omega_c t})
\end{aligned}
$$

- 系统有一个守恒量

$$\hat{N} = \hat{a}^\dagger \hat{a} + |e\rangle\langle e|$$

- 子空间 $\{|e, n-1\rangle, |g, n\rangle$

$$H_n = \begin{pmatrix} \varepsilon_e + (n-1)\hbar\omega_c & g\sqrt{n} \\ g\sqrt{n} & \varepsilon_g + n\hbar\omega_c \end{pmatrix}$$

- 本征值 $(\Delta = \varepsilon_e - \varepsilon_g - \hbar\omega_c)$

$$E_{n,\pm} = n\hbar\omega_c + \frac{\varepsilon_e + \varepsilon_g - \hbar\omega_c}{2} \pm \frac{1}{2}\sqrt{\Delta^2 + 4g^2 n}$$

量子相关函数

nature photonics

REVIEW ARTICLE

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Solid-state single-photon emitters

Igor Aharonovich[1,2*], Dirk Englund[3] and Milos Toth[1,2]

Single-photon emitters play an important role in many leading quantum technologies. There is still no 'ideal' on-demand single-photon emitter, but a plethora of promising material systems have been developed, and several have transitioned from proof-of-concept to engineering efforts with steadily improving performance. Here, we review recent progress in the race towards true single-photon emitters required for a range of quantum information processing applications. We focus on solid-state systems including quantum dots, defects in solids, two-dimensional hosts and carbon nanotubes, as these are well positioned to benefit from recent breakthroughs in nanofabrication and materials growth techniques. We consider the main challenges and key advantages of each platform, with a focus on scalable on-chip integration and fabrication of identical sources on photonic circuits.

量子相关函数

Table 1 | Summary of photophysical properties of solid-state SPEs.

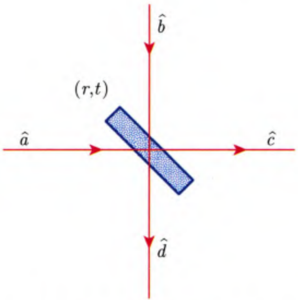
	Maximum count rate (without a cavity, continuous wave) (counts s^{-1})	Lifetime (ns)	Homogeneous linewidth at 4 K	Indistinguishable photons (IP) and entanglement (E)	Spatial targeted fabrication of single emitters	Operation temperature	Integration of SPEs with dielectric cavities or plasmonic resonators
Colour centres in diamond	SiV: ~3×10^6 (ref. 138)* NV: ~1×10^6 (ref. 139)† For other sources see ref. 28	SiV: ~1 NV: ~12–22	NV, SiV lifetime-limited[29,30] Cr-related: 4 GHz (ref. 140)	NV: IP, E SiV: IP	Only for NV and SiV (ref. 28)	RT	Dielectric: NV, SiV only Plasmonics: NV only
Defects in SiC, ZnO and BN Rare earths in YAG/YOS	YAG: ~60×10^3 (ref. 141) ZnO: ~1×10^5 (ref. 44) SiC: ~2×10^6 (ref. 35) BN: ~3×10^6 (ref. 56)	19 (ref. 49)§ 1–4 (ref. 44) 1–4 (ref. 35) ~3 (ref. 56)	N/A	No	No	RT	No
Arsenide QDs	~1×10^7 (ref. 84)‡	~1 (refs 6,84)	Lifetime-limited	Yes	Yes	4 K	Yes
Nitride QDs	N/A	~0.3 (ref. 85)	~1.5 meV (ref. 85)	No	Yes	RT	Dielectric: yes Plasmonics: no
CNTs	~3×10^3 (ref. 68)	~0.4 (ref. 68)	N/A	No	No	RT	Dielectric: yes Plasmonics: no
2D TMDCs	~3.7×10^5 (ref. 57)	~1–3 (ref. 57)	N/A	No	No	4 K	No

The reported count rates for each system can be potentially optimized by integrating with cavities or improving collection optics. *Reported from a nanodiamond on iridium. †Recorded from a nanodiamond positioned on a solid immersion lens. Similar values obtained by etching a bullseye grating into a diamond membrane[142]. In both cases emitters in bulk diamond are dimmer. ‡Count rate at the objective, which is directly comparable to other systems. §Realized by optical upconversion to a short-lived excited state. N/A, not available; RT, room temperature.

单光子源

- 量子力学基础原理验证
- 量子通信：量子密钥分发，光子作为信息载体
  量子不可克隆，防止窃听
- 量子计算
- 精密测量

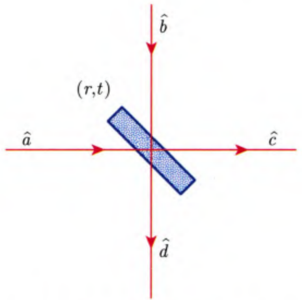
分束器 (beam splitter)



$$\boldsymbol{E}_{\text{out}} = \begin{pmatrix} E_c \\ E_d \end{pmatrix} = \begin{pmatrix} r_{ac} & t_{bc} \\ t_{ad} & r_{bd} \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix} = \tau \boldsymbol{E}_{\text{in}}$$

- 转移矩阵：每个元素都是复数
- t_{mn} 表示穿透系数，r_{mn} 表示反射系数

分束器 (beam splitter)



$$\boldsymbol{E}_{\text{out}} = \begin{pmatrix} E_c \\ E_d \end{pmatrix} = \begin{pmatrix} r_{ac} & t_{bc} \\ t_{ad} & r_{bd} \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix} = \tau \boldsymbol{E}_{\text{in}}$$

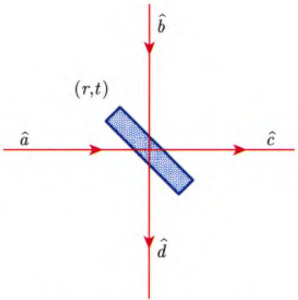
能量守恒

$$|E_c|^2 + |E_d|^2 = |E_a|^2 + |E_b|^2$$

得到

$$r_{ac}r_{ac}^* + t_{ad}t_{ad}^* = 1, \quad r_{bd}r_{bd}^* + t_{bc}t_{bc}^* = 1, \quad r_{ac}t_{bc}^* + t_{ad}r_{bd}^* = 0$$

分束器 (beam splitter)



$$\boldsymbol{E}_{\text{out}} = \begin{pmatrix} E_c \\ E_d \end{pmatrix} = \begin{pmatrix} r_{ac} & t_{bc} \\ t_{ad} & r_{bd} \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix} = \tau \boldsymbol{E}_{\text{in}}$$

转移矩阵 τ 是幺正的

$$\tau^\dagger \tau = I$$

分束器

设 $|t_{ad}| = |t_{bc}| = T$, $|r_{ac}| = |r_{bd}| = R$, $R^2 + T^2 = 1$

$$\begin{pmatrix} E_c \\ E_d \end{pmatrix} = \begin{pmatrix} Re^{i\phi_{ac}} & Te^{i\phi_{bc}} \\ Te^{i\phi_{ad}} & Re^{i\phi_{bd}} \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$

- 同时需要满足

$$RTe^{i(\phi_{ac}-\phi_{bc})} + RTe^{i(\phi_{ad}-\phi_{bd})} = 0$$

即 $\phi_{ad} - \phi_{bd} + \phi_{bc} - \phi_{ac} = \pi$

- 六个未知数，两个限制条件
- 定义 $T = \cos\theta$, $R = \sin\theta$

$$\hat{\tau}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

or

$$\hat{\tau}(\theta) = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix} = e^{i\theta\sigma_x}$$

分束器

设 $|t_{ad}| = |t_{bc}| = T$，$|r_{ac}| = |r_{bd}| = R$，$R^2 + T^2 = 1$

$$\begin{pmatrix} E_c \\ E_d \end{pmatrix} = \begin{pmatrix} Re^{i\phi_{ac}} & Te^{i\phi_{bc}} \\ Te^{i\phi_{ad}} & Re^{i\phi_{bd}} \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$

- 同时需要满足

$$RTe^{i(\phi_{ac}-\phi_{bc})} + RTe^{i(\phi_{ad}-\phi_{bd})} = 0$$

即 $\phi_{ad} - \phi_{bd} + \phi_{bc} - \phi_{ac} = \pi$

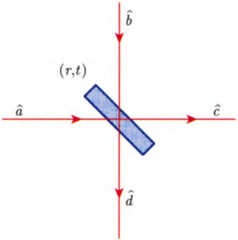
- 对于半反半透分束器，$\theta = \pi/4$，一种取法

$$\tau = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

另一种常见取法

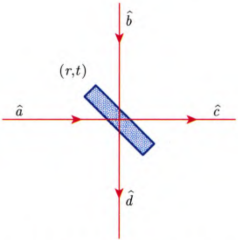
$$\tau = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

分束器的量子力学描述



- 量子力学中，电磁场用算符表示
- 不同端口也可以用对应的产生湮灭算符表示
- 把光通过分束器理解成一个演化过程
- ρ_{in} 表示初始时刻的态
- ρ_{out} 表示离开分束器的态

分束器的量子力学描述



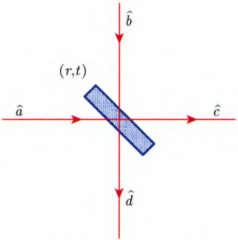
- 光场通过分束器，可以看成一个幺正演化过程
- 等效哈密顿量

$$\begin{pmatrix}\hat{c}\\ \hat{d}\end{pmatrix}_{\text{out}} = \hat{\tau}(\theta)\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}_{\text{in}} = \hat{S}^{\dagger}(\theta)\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}_{\text{in}}\hat{S}(\theta) = \begin{pmatrix}\hat{a}(\theta)\\ \hat{b}(\theta)\end{pmatrix}_{\text{in}}$$

其中 $\hat{a}(\theta) = \hat{a}\cos\theta + i\hat{b}\sin\theta$, $\hat{b}(\theta) = \hat{b}\cos\theta + i\hat{a}\sin\theta$

$$\hat{S}(\theta) = \exp\left[i\theta(\hat{a}^{\dagger}\hat{b} + \hat{b}^{\dagger}\hat{a})\right]$$

分束器的量子力学描述



- **海森堡表象**: $\hat{a}, \hat{b} \to \hat{c}, \hat{d}$; $\rho_{\rm in} \to \rho_{\rm in}$

$$\mathrm{Tr}\left[\rho_{\rm in} f(\hat{c}, \hat{d})\right]$$

- **薛定谔表象**: $\hat{a}, \hat{b} \to \hat{a}, \hat{b}$; $\rho_{\rm in} \to \rho_{\rm out}$

$$\mathrm{Tr}\left[\rho_{\rm out} f(\hat{a}, \hat{b})\right]$$

- **两个表象的期望应该相同**

$$\rho_{\rm out} = \hat{S}(\theta)\rho_{\rm in}\hat{S}^\dagger(\theta), \quad \mathrm{Tr}\left[\rho_{\rm in} f(\hat{c}, \hat{d})\right] = \mathrm{Tr}\left[\rho_{\rm out} f(\hat{a}, \hat{b})\right]$$

分束器

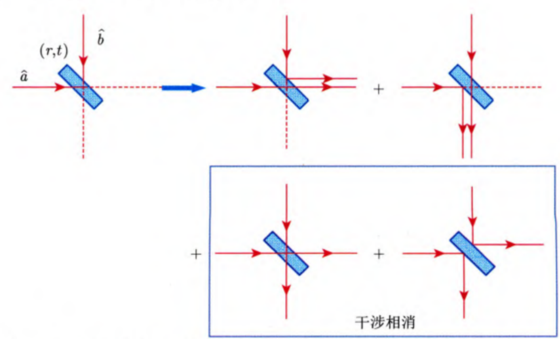
例子：单光子从端口 a 入射

$$
\begin{aligned}
|\psi\rangle_{\text{out}} &= \hat{S}(\theta)\,|1,0\rangle = \hat{S}(\theta)\hat{a}\,|0,0\rangle \\
&= \hat{S}(\theta)\hat{a}\hat{S}^{\dagger}(\theta)\hat{S}(\theta)\,|0,0\rangle \\
&= (\cos\theta\hat{a}^{\dagger} + i\sin\theta\hat{b}^{\dagger})\,|0,0\rangle \\
&= \cos\theta\,|1,0\rangle + i\sin\theta\,|0,1\rangle
\end{aligned}
$$

分束器

例子：两个端口分别有一个光子入射

$$
\begin{aligned}
|\psi\rangle_{\text{out}} &= \hat{S}(\theta)\,|1,1\rangle = \hat{S}(\theta)\hat{a}^\dagger\hat{S}^\dagger(\theta)\hat{S}(\theta)\hat{b}^\dagger\hat{S}^\dagger(\theta)\,|0,0\rangle \\
&= (\cos\theta\hat{a}^\dagger + i\sin\theta\hat{b}^\dagger)\otimes(\cos\theta\hat{b}^\dagger + i\sin\theta\hat{a}^\dagger)\,|0,0\rangle \\
&= \frac{1}{\sqrt{2}}\cos\theta(|2,0\rangle - |0,2\rangle) + \cos 2\theta\,|1,1\rangle
\end{aligned}
$$



- $\theta = \pi/4$，在端口 c 和 d 做复合计数，看不到信号
- Hong-Ou-Mandel dip

分束器

- **两个端口有大量光子输入**：$|N,N\rangle = (\hat{a}^\dagger)^N (\hat{b}^\dagger)^N |0,0\rangle / N!$

$$
\begin{aligned}
|\psi\rangle_{\text{out}} = \hat{S}(\theta) &= \frac{1}{N!}\left(\frac{\hat{a}^\dagger + i\hat{b}^\dagger}{\sqrt{2}}\right)^N \left(\frac{i\hat{a}^\dagger + \hat{b}^\dagger}{\sqrt{2}}\right)^N |0,0\rangle \\
&= \frac{i^N}{N!2^N}\left(\hat{a}^{\dagger 2} + \hat{b}^{\dagger 2}\right)^N |0,0\rangle \\
&= \frac{i^N}{N!2^N}\sum_{m=0}^{N} C_N^m \sqrt{(2m)!(2N-2m)!}\,|2m, 2N-2m\rangle
\end{aligned}
$$

分束器

当分束器 a 端口输入相干态，系统输入即为 $|\alpha, 0\rangle = \hat{D}(\alpha)|0,0\rangle$:

$$
\begin{aligned}
|\psi\rangle_{\text{out}} &= \hat{S}(\theta)|\alpha, 0\rangle = \hat{S}(\theta)\hat{D}(\alpha)\hat{S}^\dagger(\theta)|0,0\rangle \\
&= \exp\left[\alpha(\hat{a}^\dagger \cos\theta + i\sin\theta \hat{b}) - \text{H. c.}\right]|0,0\rangle \\
&= |\alpha\cos\theta, i\alpha\sin\theta\rangle
\end{aligned}
$$

分束器

当分束器输入双模压缩相干态时，系统输出

$$\begin{aligned}|\psi\rangle_{\text{out}} &= \hat{S}(\theta)\exp\left[\xi\hat{a}^\dagger\hat{b}^\dagger - \xi^*\hat{a}\hat{b}\right]|0,0\rangle \\ &= \exp\left[\xi(\hat{a}^\dagger(-\theta)\hat{b}^\dagger(-\theta)) - \text{H. c.}\right]|0,0\rangle\end{aligned}$$

其中

$$\begin{aligned}\hat{a}(-\theta)\hat{b}(-\theta) &= (\hat{a}\cos\theta - i\sin\theta\hat{b})(\hat{b}\cos\theta - i\sin\theta\hat{a}) \\ &= \cos(2\theta)\hat{a}\hat{b} - i\frac{1}{2}\sin(\theta)(\hat{a}^2 + \hat{b}^2)\end{aligned}$$

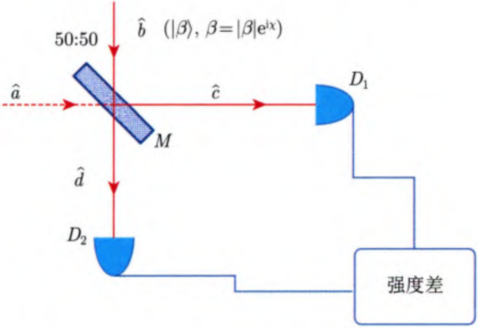
如果 $\theta = \pi/4$

$$|\psi\rangle_{\text{out}} = \exp\left[i\frac{1}{2}(\xi\hat{a}^{\dagger 2} + \xi^*\hat{a}^2)\right]|0\rangle \exp\left[i\frac{1}{2}(\xi\hat{b}^{\dagger 2} + \xi^*\hat{b}^2)\right]|0\rangle$$

双模压缩态经过半透半反分束器后变成两个单模压缩态直积

压缩光场的平衡零拍探测

Balanced Homodyne detection



- $\hat{c} = \frac{\hat{a}+i\hat{b}}{\sqrt{2}}, \quad \hat{d} = \frac{\hat{b}+i\hat{a}}{\sqrt{2}}$
- 输出 $\hat{c}$ 和 $\hat{d}$ 的强度差

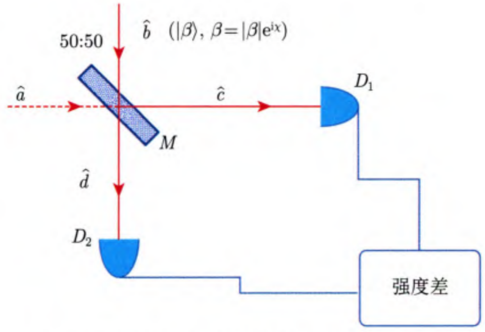
$$\hat{n}_{cd} = \hat{c}^\dagger \hat{c} - \hat{d}^\dagger \hat{d} = i(\hat{a}^\dagger \hat{b} - \hat{b}^\dagger \hat{a})$$

- $\hat{b}$ 输入为相干态 $\beta = |\beta| e^{i\chi}$

$$\langle \hat{n}_{cd} \rangle = |\beta| \left\langle \hat{a}^\dagger e^{i(\chi+\pi/2)} + \hat{a} e^{-i(\chi+\pi/2)} \right\rangle = 2|\beta| \langle X(\theta) \rangle$$

压缩光场的平衡零拍探测

Balanced Homodyne detection

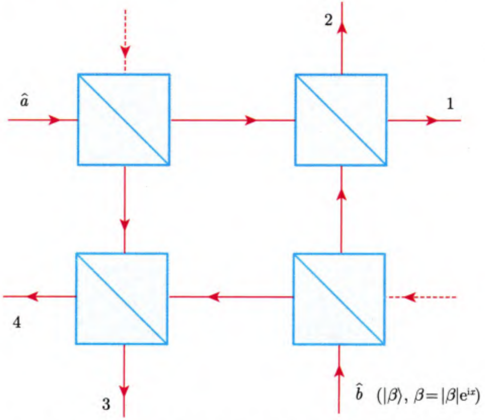


- 通过改变本地相干态相位 χ，就可以改变 θ，从而测量光场任意方向的正交分量

$$(\Delta n_{cd})^2 = 4|\beta|^2(\Delta X(\theta))^2$$

- 对角度进行扫描就可以画出压缩态 Q 函数的椭圆

压缩光场的平衡零拍探测

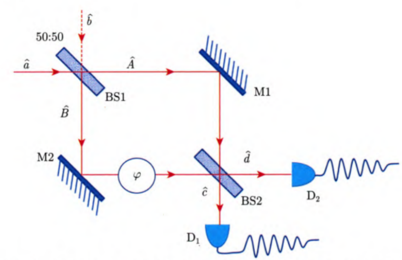


- 对装置进行扩展，形成八个端口

$$\langle \hat{n}_{12}\rangle = 2|\beta|\left\langle \hat{X}(\chi+\pi/2)\right\rangle, \quad \langle \hat{n}_{34}\rangle = 2|\beta|\left\langle \hat{X}(\chi)\right\rangle$$

第三章

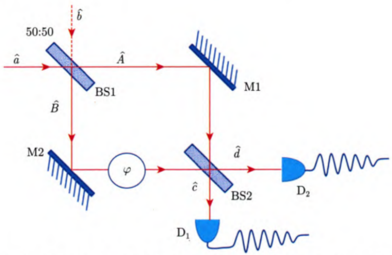
Mach-Zehnder 干涉仪



$$\begin{pmatrix}\hat{c}\\ \hat{d}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{i\varphi}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}$$

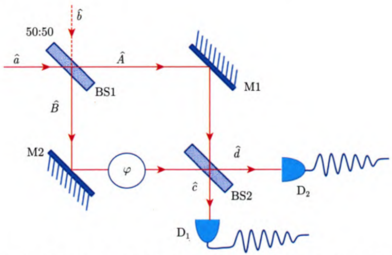
$$= e^{i(\varphi/2+\pi/2)}\begin{pmatrix}-\sin\frac{\varphi}{2}\hat{a} + \cos\frac{\varphi}{2}\hat{b}\\ \cos\frac{\varphi}{2}\hat{a} + \sin\frac{\varphi}{2}\hat{b}\end{pmatrix}$$

Mach-Zehnder 干涉仪



- 当 $\varphi = 0$ 时： $\hat{c} = i\hat{b}$, $\hat{d} = i\hat{a}$
- 当 $\varphi = \pi$ 时： $\hat{c} = \hat{a}$, $\hat{d} = -\hat{b}$

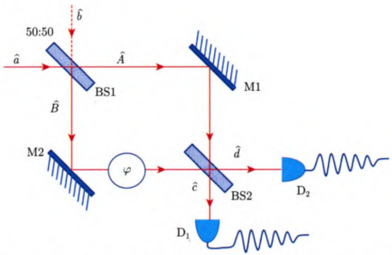
Mach-Zehnder 干涉仪



$$\hat{c}^\dagger\hat{c} = \sin^2\frac{\varphi}{2}\hat{a}^\dagger\hat{a} + \cos^2\frac{\varphi}{2}\hat{b}^\dagger\hat{b} - \cos\frac{\varphi}{2}\sin\frac{\varphi}{2}(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)$$

$$\hat{d}^\dagger\hat{d} = \cos^2\frac{\varphi}{2}\hat{a}^\dagger\hat{a} + \sin^2\frac{\varphi}{2}\hat{b}^\dagger\hat{b} + \cos\frac{\varphi}{2}\sin\frac{\varphi}{2}(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)$$

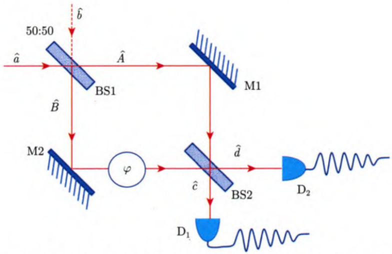
Mach-Zehnder 干涉仪



输出端口光子数差

$$\hat{n}_{cd} = \hat{c}^\dagger \hat{c} - \hat{d}^\dagger \hat{d} = (\hat{b}^\dagger \hat{b} - \hat{a}^\dagger \hat{a}) \cos\varphi - (\hat{a}^\dagger + \hat{a}\hat{b}^\dagger) \sin\varphi$$

Mach-Zehnder 干涉仪

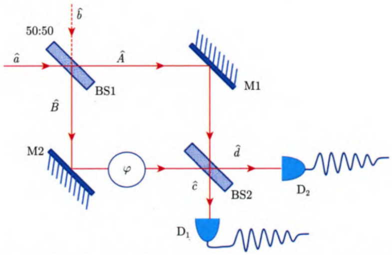


如果 $\hat{b}$ 模式输入相干光场 $\left\langle\hat{b}\right\rangle = \beta = |\beta|e^{i\phi}$:

$$\langle\hat{n}_{cd}\rangle\Big|_{\varphi=\pi/2} = -|\beta|\left\langle\hat{a}^\dagger e^{i\phi} + \hat{a}e^{-i\phi}\right\rangle = -2|\beta|\left\langle\hat{X}_\phi\right\rangle$$

即前面的平衡零拍探测

Mach-Zehnder 干涉仪



假定输入端 $\hat{a}$ 为真空态：

$$\left\langle \hat{c}^\dagger \hat{c} \right\rangle = n_l \cos^2 \frac{\varphi}{2}, \quad \left\langle \hat{d}^\dagger \hat{d} \right\rangle = n_l \sin^2 \frac{\varphi}{2}, \quad \langle \hat{n}_{cd} \rangle = n_l \cos \varphi$$

- 其中 $n_l = |\beta|^2$ 为相干场的平均光子数
- 可以利用干涉条纹来测量光路中相位移动 φ

Mach-Zehnder 干涉仪

$\hat{n}_{cd}$ 具有很强的量子特性

$$(\Delta n_{cd})^2\Big|_{\varphi=\pi/2} = \left\langle(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)\right\rangle = |\beta|^2 = n_l$$

探测必然存在误差

$$\left|\frac{\partial\langle\hat{n}_{cd}\rangle}{\partial\varphi}\right| = n_l \sin\varphi$$

表明：干涉条纹在 $\varphi = \pi/2$ 时变化最剧烈

$$\Delta\varphi = \frac{\Delta n_{cd}}{|\partial\langle\hat{n}_{cd}\rangle/\partial\varphi|} = \frac{\sqrt{n_l}}{n_l} = \frac{1}{\sqrt{n_l}}$$

- 测量误差与光场强度成反比
- 误差极限 $\frac{1}{\sqrt{n_l}}$ 称为标准量子极限 (standard quantum limit)

Mach-Zehnder 干涉仪

为更清楚说明问题，这里在 $\varphi = \pi/2$ 附近展开

$$\varphi = \pi/2 + \Delta\varphi, \quad \hat{a} = \delta\hat{a}_1 + i\delta\hat{a}_2, \quad \hat{b} = \beta + \delta\hat{b}_1 + i\delta\hat{b}_2$$

得到

$$\begin{aligned}
\hat{c}^\dagger\hat{c} &= \frac{1+\sin\Delta\varphi}{2}\hat{a}^\dagger\hat{a} + \frac{1-\sin\Delta\varphi}{2}\hat{b}^\dagger\hat{b} - \frac{1}{2}\cos\Delta\varphi(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger) \\
&= \frac{1-\Delta\varphi}{2}\beta^2 + \beta(\delta\hat{b}_1 - \delta\hat{a}_1) \\
\hat{d}^\dagger\hat{d} &= \frac{1-\sin\Delta\varphi}{2}\hat{a}^\dagger\hat{a} + \frac{1+\sin\Delta\varphi}{2}\hat{b}^\dagger\hat{b} + \frac{1}{2}\cos\Delta\varphi(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger) \\
&= \frac{1+\Delta\varphi}{2}\beta^2 + \beta(\delta\hat{b}_1 + \delta\hat{a}_1)
\end{aligned}$$

测量信号 (设 β 为实数)

$$\hat{n}_{cd} \approx -\beta^2\Delta\varphi - 2\beta\delta\hat{a}_1$$

- 信号信息 $\geq$ 噪声信息

$$(\beta^2\Delta\varphi)^2 \geq 4\beta^2\left\langle\delta\hat{a}_1^2\right\rangle$$

Mach-Zehnder 干涉仪

探测的最小相位满足

$$\Delta\varphi_{\min} \approx \left(\frac{4\left\langle\delta\hat{a}_1^2\right\rangle}{\beta^2}\right)^{1/2} = \frac{1}{\sqrt{n_l}}$$

Mach-Zehnder 干涉仪

探测的最小相位满足

$$\Delta\varphi_{\min} \approx \left(\frac{4\left\langle\delta\hat{a}_1^2\right\rangle}{\beta^2}\right)^{1/2} = \frac{1}{\sqrt{n_l}}$$

- 标准量子极限来源输入端 $\hat{a}$ 的真空涨落
- 为提高测量精度，可以降低 $\hat{a}$ 输入端的真空涨落噪声
- $\hat{a}$ 输入真空压缩态 $|0,r\rangle$: $\left\langle\delta\hat{a}_1^2\right\rangle = e^{-2r}/4$, 平均光子数 $n_a = \sinh^2 r$

$$\hat{n}_{cd} \approx (n_a - \beta^2)\Delta\varphi - 2\beta\delta\hat{a}_1$$

从而有

$$\Delta\varphi_{\min} \approx \left(\frac{4\beta^2\left\langle\delta\hat{a}_1^2\right\rangle}{(n_a-\beta^2)^2}\right)^{1/2} \approx \frac{e^{-r}}{\sqrt{n_l}}$$

当 r 不是很大时 $\beta \gg n_a$

振福压缩态

- 压缩相干态：某个方向的正交相位分量具有低于真空起伏的噪声
- 光场的振福和相位是一对共轭物理量
- 振幅压缩态：$\left\langle (\Delta \hat{N})^2 \right\rangle < \left\langle \hat{N} \right\rangle$
- 只需要光子计数器
- 光通信、信息处理、以及精密探测和原子光谱学

振幅压缩态

$$\hat{N} = \hat{a}^\dagger \hat{a},$$

$$\hat{S} = \frac{1}{2i}\left[\frac{1}{\sqrt{\hat{N}+1}}\hat{a} - \hat{a}^\dagger \frac{1}{\sqrt{\hat{N}+1}}\right],$$

$$\hat{C} = \frac{1}{2}\left[\frac{1}{\sqrt{\hat{N}+1}}\hat{a} + \hat{a}^\dagger \frac{1}{\sqrt{\hat{N}+1}}\right]$$

满足对易关系

$$\left[\hat{N}, \hat{C}\right] = -i\hat{S}, \quad \left[\hat{N}, \hat{S}\right] = i\hat{C}$$

不确定性关系

$$\left\langle \Delta\hat{N}^2 \right\rangle \left\langle \Delta\hat{S}^2 \right\rangle \geq \frac{1}{4}\left\langle \hat{C} \right\rangle^2, \quad \left\langle \Delta\hat{N}^2 \right\rangle \left\langle \Delta\hat{C}^2 \right\rangle \geq \frac{1}{4}\left\langle \hat{S} \right\rangle^2,$$

- **当电磁场激发数很大时** $\rightarrow \left\langle \hat{C} \right\rangle \sim 1$

振幅压缩态

最小不确定态

$$(\hat{N} - \left\langle \hat{N} \right\rangle) \left| \psi \right\rangle = \lambda (\hat{S} - \left\langle \hat{S} \right\rangle) \left| \psi \right\rangle$$

也就是

$$\left(\hat{N} - \lambda \hat{S} \right) = \left(\left\langle \hat{N} \right\rangle - \lambda \left\langle \hat{S} \right\rangle \right) \left| \psi \right\rangle$$

振幅压缩态

最小不确定态

$$(\hat{N} - \left\langle \hat{N} \right\rangle) |\psi\rangle = \lambda(\hat{S} - \left\langle \hat{S} \right\rangle) |\psi\rangle$$

也就是

$$\left(e^{-r}\hat{N} + ie^{r}\hat{S}\right) = \left(e^{-r}\left\langle \hat{N} \right\rangle + ie^{r}\left\langle \hat{S} \right\rangle\right) |\psi\rangle$$

振幅压缩态

涨落满足

$$\left\langle \Delta \hat{N}^2 \right\rangle = \frac{1}{2} e^{-2r}, \quad \left\langle \Delta \hat{S}^2 \right\rangle \approx \left\langle \Delta \hat{\phi}^2 \right\rangle = \frac{1}{2} e^{2r}$$

$$\left\langle \Delta \hat{N}^2 \right\rangle \left\langle \Delta \hat{S}^2 \right\rangle \approx \left\langle \Delta \hat{N}^2 \right\rangle \left\langle \Delta \hat{\phi}^2 \right\rangle = \frac{1}{4}$$

振幅压缩态

当 $r = -\frac{1}{2}\ln\left(2\left\langle\hat{N}\right\rangle\right)$

$$\left\langle\Delta\hat{N}^2\right\rangle = \left\langle\hat{N}\right\rangle, \quad \left\langle\Delta\hat{S}^2\right\rangle \approx \left\langle\Delta\hat{\phi}^2\right\rangle = \frac{1}{4\left\langle\hat{N}\right\rangle}$$

- 粒子数涨落与相干态一致，但不是相干态

振幅压缩态

如何生成粒子数-相位压缩态?

振幅压缩态

单模光场通过非线性介质

$$\hat{H}_I = \hbar\chi\hat{a}^{\dagger 2}\hat{a}^2 = \hbar\chi\hat{N}(\hat{N}-1)$$

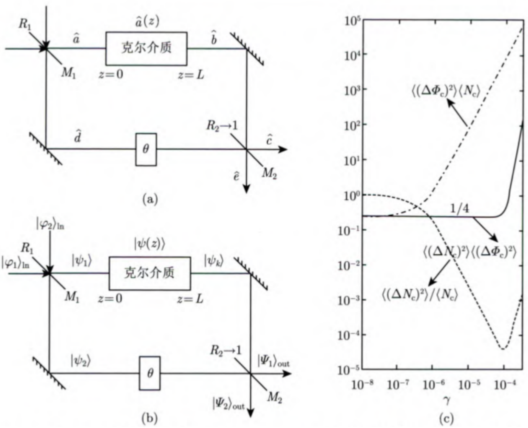
- 演化幺正算符

$$\hat{U}_K(L) = \exp\left[i\frac{\gamma}{2}\hat{N}(\hat{N}-1)\right]$$

- 场算符

$$\hat{b} = \hat{U}_K^\dagger(L)\hat{a}\hat{U}_K(L) = e^{i\gamma\hat{N}}\hat{a}$$

振幅压缩态

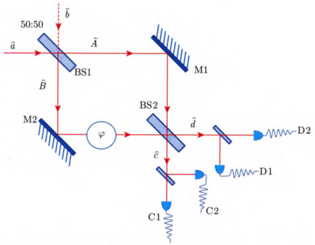


R_1
$\hat{a}(z)$
$\hat{a}$
克尔介质
$\hat{b}$
M_1
$z=0$
$z=L$
$R_2\to 1$
$\hat{d}$
θ
$\hat{c}$
$\hat{e}$
M_2
(a)
$|\varphi_2\rangle_{\rm in}$
R_1
$|\psi(z)\rangle$
$|\varphi_1\rangle_{\rm in}$
$|\psi_1\rangle$
克尔介质
$|\psi_k\rangle$
M_1
$z=0$
$z=L$
$R_2\to 1$
$|\psi_2\rangle$
θ
$|\Psi_1\rangle_{\rm out}$
$|\Psi_2\rangle_{\rm out}$
M_2
(b)
$\langle(\Delta\Phi_c)^2\rangle\langle N_c\rangle$
1/4
$\langle(\Delta N_c)^2\rangle\langle(\Delta\Phi_c)^2\rangle$
$\langle(\Delta N_c)^2\rangle/\langle N_c\rangle$
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4}
γ
(c)

多光子干涉仪

端口 a 和 b 各输入一个光子，端口 c 和 d 做符合计数：

$$\langle 1_a, 1_b|\hat{c}^\dagger \hat{d}^\dagger \hat{d}\hat{c}|1_a, 1_b\rangle = \frac{1}{2}(1+\cos 2\varphi)$$

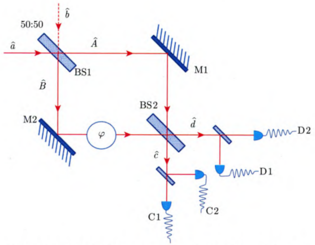


- 端口 A 和 B: $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|2_a, 0_b\rangle + |0_a, 2_b\rangle)$
- 经过相位片后: $|\psi'_{AB}\rangle = \frac{1}{\sqrt{2}}(|2_a, 0_b\rangle + e^{i2\varphi}\,|0_a, 2_b\rangle)$

多光子干涉仪

端口 a 和 b 各输入一个光子，端口 c 和 d 做符合计数:

$$\langle 1_a, 1_b|\hat{c}^\dagger \hat{d}^\dagger \hat{d}\hat{c}|1_a, 1_b\rangle = \frac{1}{2}(1 + \cos 2\varphi)$$



- 经过 BS2 后

$$S(\theta = \frac{\pi}{4})\,|\psi'_{AB}\rangle = \frac{1}{2}S(\frac{\pi}{4})[(\hat{a}^\dagger)^2 + e^{i2\varphi}(\hat{b}^\dagger)^2]\,|0_a, 0_b\rangle$$

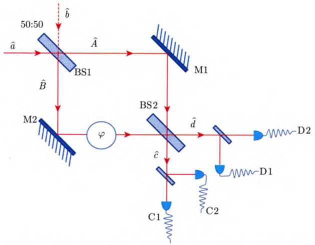
$$= \frac{1}{2}\left[\left(\frac{\hat{a}^\dagger + i\hat{b}^\dagger}{\sqrt{2}}\right)^2 + e^{i2\varphi}\left(\frac{i\hat{a}^\dagger + \hat{b}^\dagger}{\sqrt{2}}\right)^2\right]|0_a, 0_b\rangle$$

$$= \frac{i}{2}(1 + e^{i2\varphi})\hat{a}^\dagger\hat{b}^\dagger\,|0_a, 0_b\rangle + \frac{1 - e^{i2\varphi}}{4}[(\hat{a}^\dagger)^2 - (\hat{b}^\dagger)^2]\,|0_a, 0_b\rangle$$

多光子干涉仪

端口 a 和 b 各输入一个光子，端口 c 和 d 做符合计数：

$$\langle 1_a, 1_b|\hat{c}^\dagger \hat{d}^\dagger \hat{d}\hat{c}|1_a, 1_b\rangle = \frac{1}{2}(1+\cos 2\varphi)$$



- 符合计数率

$$P_{cd} = \frac{1}{4}\left|1+e^{i2\varphi}\right|^2 = \frac{1}{2}\left(1+\cos 2\varphi\right)$$

- 当 $\varphi = \pi/2$ 时，$P_{cd} = 0$

$$|\psi_{cd}\rangle = \frac{1}{\sqrt{2}}\left(|2_a, 0_b\rangle - |0_a, 2_b\rangle\right)$$

多光子干涉仪

4 光子入射态 $|2_a, 2_b\rangle$:

- 经过 BS1 后:

$$\hat{A} = \sqrt{T}\hat{a} + i\sqrt{R}\hat{b}, \quad \hat{B} = \sqrt{T}\hat{b} + i\sqrt{R}\hat{a}$$

- BS2 为半透半反的分束器，经过 BS2 后:

$$\hat{c} = \frac{1}{\sqrt{2}}\left(\hat{A} + e^{i\varphi} i\hat{B}\right), \quad \hat{d} = \frac{1}{\sqrt{2}}\left(i\hat{A} + e^{i\varphi}\hat{B}\right)$$

- 输出端做 4 光子符合计数，计数概率为

$$\begin{aligned} P_{cd} &= \langle 2_a, 2_b|(\hat{c}^\dagger)^2(\hat{d}^\dagger)^2\hat{d}^2\hat{c}^2|2_a, 2_b\rangle \\ &\propto \left|e^{i2\varphi}(T^2 + R^2 - 4TR) - 3TR(1 + e^{i4\varphi})\right|^2 \end{aligned}$$

当 φ 变化 2π，干涉条纹变化 4 个周期

NOON 态和海森堡极限

- 粒子数和相位是共轭的物理量，满足 $\left[\hat{n}, \hat{\phi}\right] = i$

$$\Delta n \Delta \phi \geq \frac{1}{2}$$

- 一般来说，粒子数涨落不会超过系统的总粒子数 $\Delta n \leq N$，得到

$$\Delta \phi \geq \frac{1}{2\Delta n} \sim \frac{1}{N} \rightarrow \quad \text{海森堡极限}$$

- 标准量子极限 $1/\sqrt{N}$

第三章

Summary

- 相干度