量子光学
第六章 激光冷却技术

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课程大纲

量子光学是研究光场的量子性质，及其与原子、分子等物质相互作用的物理学分支，核心内容包括光子的产生、操控与探测以及光–物质的量子相干与纠缠现象。

第一章 辐射场及其量子化 (2 次课)

第二章 **量子相干态** (3 次课)

第三章 **光场相干性及其干涉** (3 次课)

第四章 **光场与原子相互作用** (4 次课)

第五章 热库系统与主方程 (2 次课)

第六章 激光冷却技术 (2 次课)

第七章 光学腔和光力耦合系统 (2 次课)

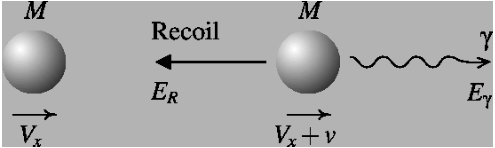
参考资料

- 《量子光学》，郭光灿，周祥发，科学出版社
- Quantum Optics, Marlan O. Scully, M. Suhail Zubairy, Cambridge University Press
- 维基百科 (wikipedia.org)
- Atom Optics, Pierre Meystre
- Notes from Prof. Wei Yi

Laser cooling

激光冷却是利用激光与原子之间的动量交换来降低原子质心运动动能的技术。

- 光子携带动量 $\hbar \boldsymbol{k}$
- 原子吸收 $\rightarrow$ 自发辐射 $\rightarrow$ 动量转移



Laser cooling

- Doppler cooling
- Sisyphus cooling
- Raman cooling
- Evaporative cooling
- Sideband cooling

不同技术特点:

技术	温度	物理极限	极限公式	适用对象
Doppler 冷却	$\sim 100\ \mu$K	Doppler 极限	$T_D = \hbar\Gamma/2k_B$	中性原子
Sisyphus 冷却	$\sim$ 1–10 μK	反冲极限	$T_R = \hbar^2k^2/Mk_B$	碱金属原子
Raman 冷却	$\sim$ 10–100 nK	低于反冲极限	$T \ll T_R$	中性原子、光晶格
边带冷却	$\langle n\rangle \approx 0$	基态冷却	$\langle n\rangle \sim (\Gamma/\nu)^2$	束缚离子、光晶格原子
蒸发冷却	$\sim$ 1–100 nK	碰撞率/三体损失	$T \sim T_c/\gamma_{\mathrm{el}}$	中性原子、分子

温度 T 由原子速度的均方根确定

$$\frac{1}{2}k_BT = \frac{1}{2}m\left\langle v^2\right\rangle$$

$^{87}\mathrm{Rb}$ 原子，温度 $T = 300\,\mathsf{K}$

$$\begin{aligned}\sqrt{\langle v^2\rangle} &\sim \sqrt{\frac{3k_BT}{mc^2}c^2} = \sqrt{\frac{1.38\times 10^{-5}\times 300}{87\times 938\times 10^6}}3\times 10^8\mathrm{m/s} \\ &= 292.46\,\mathsf{m/s}\end{aligned}$$

$^{87}\mathrm{Rb}$ 原子，温度 $T = 1\,\mathsf{mK}$

$$\begin{aligned}\sqrt{\langle v^2\rangle} &\sim \sqrt{\frac{3k_BT}{mc^2}c^2} = \sqrt{\frac{1.38\times 10^{-5}\times 10^{-3}}{87\times 938\times 10^6}}3\times 10^8\mathrm{m/s} \\ &= 0.53\,\mathsf{m/s}\end{aligned}$$

JC model of two-level system:

$$H = \hbar\omega_0 |e\rangle\langle e| + \left[\frac{\hbar\Omega}{2} e^{i\omega t + i\phi(\boldsymbol{r})} |g\rangle\langle e| + \mathrm{H.\,c.}\right]$$

Laser cooling

$$\dot{\rho}_{ee} = -i\frac{\Omega}{2}\left(e^{-i\omega t - i\phi}\rho_{ge} - c.c.\right) - \Gamma\rho_{ee}$$

$$\dot{\rho}_{ge} = \left(i\omega_0 - \frac{\Gamma}{2}\right)\rho_{ge} - i\frac{\Omega}{2}e^{i\omega t + i\phi}\left(\rho_{ee} - \rho_{gg}\right)$$

$$\dot{\rho}_{gg} = i\frac{\Omega}{2}\left(e^{-i\omega t - i\phi}\rho_{ge} - c.c.\right) + \Gamma\rho_{ee}$$

Solve in the rotating frame $\rho_{ge} = \tilde{\rho}e^{i\omega t}$, steady state

$$\tilde{\rho}_{eg} = -i\frac{\Omega}{2}e^{-i\phi}\frac{\Gamma/2 + i\delta}{\delta^2 + \Gamma^2/4 + \Omega^2/2}$$

$$\tilde{\rho}_{ee} = \frac{\Omega^2/4}{\delta^2 + \Gamma^2/4 + \Omega^2/2}$$

- $\Omega \to \infty$, $\rho_{gg} = \rho_{ee} = 1/2$
- Do not consider momentum of atom

Doppler cooling

- Incorporate kinetic energy $\boldsymbol{p}^2/(2m)$
- Density matrix depends on position/momentum operator

Heisenbergy equation

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{i}{\hbar}[H, \boldsymbol{p}]$$

where

$$H = \hbar\omega_0 |e\rangle\langle e| + \frac{\hbar\Omega}{2}\left[e^{i\omega t + i\phi(\boldsymbol{r})}|g\rangle\langle e| + \mathrm{H.\,c.}\right] + \frac{\boldsymbol{p}^2}{2m}$$

Due to $[H(\boldsymbol{r}), \boldsymbol{p}] = i\hbar\nabla H(\boldsymbol{r})$, we obtain

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -\frac{\hbar}{2}\left(\nabla\Omega(\boldsymbol{r})e^{i\phi(\boldsymbol{r})}\right)e^{i\omega t}|g\rangle\langle e| + \mathrm{H.\,c.}$$

Laser cooling

The force

$$\boldsymbol{F} = \left\langle \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \right\rangle = -\frac{\hbar}{2} \left\langle |e\rangle\langle g| \, \nabla \left(\Omega(\boldsymbol{r}) e^{-i\phi(\boldsymbol{r})} \right) e^{-i\omega t} + c.c. \right\rangle$$

- Spontaneous lifetime of atom
- Recoil frequency of atom $\sim$ kHz
- Atom internal state $\rightarrow$ steady state

Laser cooling

光场对原子的力

$$
\begin{aligned}
\boldsymbol{F} = \left\langle \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \right\rangle &= -\frac{\hbar}{2} \left\langle |e\rangle\langle g| \, \nabla \left(\Omega(\boldsymbol{r}) e^{-i\phi(\boldsymbol{r})} \right) e^{-i\omega t} + c.c. \right\rangle \\
&= -\frac{\hbar}{2} \langle |e\rangle\langle g| \rangle_{\text{internal}} \left\langle \nabla \left(\Omega(\boldsymbol{r}) e^{-i\phi(\boldsymbol{r})} \right) e^{-i\omega t} \right\rangle_{\text{external}} + c.c. \\
&= -\frac{\hbar}{2} \rho_{ge} \left\langle \nabla \left(\Omega(\boldsymbol{r}) e^{-i\phi(\boldsymbol{r})} \right) e^{-i\omega t} \right\rangle_{\text{external}} + c.c.
\end{aligned}
$$

- Assume Atom in steady state
- Ignore entanglement
- external parts are still complicated： classical center-of-mass

Laser cooling

$$\begin{aligned}\boldsymbol{F} &= \left\langle \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \right\rangle = -\frac{\hbar}{2}\rho_{ge}\left[(\nabla\Omega(\boldsymbol{r})) + \Omega(\boldsymbol{r})(-i)(\nabla\phi(\boldsymbol{r}))\right] e^{-i\phi(\boldsymbol{r})-i\omega t} + c.c. \\ &= F_{\mathrm{dissip}} + F_{\mathrm{react}}\end{aligned}$$

其中

$$\begin{aligned}F_{\mathrm{dissip}} &= \hbar\boldsymbol{k}\frac{\Gamma}{2}\frac{\Omega^2/2}{\delta^2+\Gamma^2/4+\Omega^2/2} \\ F_{\mathrm{react}} &= -\frac{\hbar\delta}{4}\frac{\nabla\Omega^2}{\delta^2+\Gamma^2/4+\Omega^2/2}\end{aligned}$$

- **用到了** $\phi(\boldsymbol{r}) = -\boldsymbol{k}\cdot\boldsymbol{r}$

Laser cooling

$$
\begin{aligned}
\boldsymbol{F} &= \left\langle \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \right\rangle = -\frac{\hbar}{2}\rho_{ge}\left[(\nabla\Omega(\boldsymbol{r})) + \Omega(\boldsymbol{r})(-i)(\nabla\phi(\boldsymbol{r}))\right] e^{-i\phi(\boldsymbol{r})-i\omega t} + c.c. \\
&= F_{\text{dissip}} + F_{\text{react}}
\end{aligned}
$$

其中

$$
\begin{aligned}
F_{\text{dissip}} &= \hbar\boldsymbol{k}\frac{\Gamma}{2}\frac{\Omega^2/2}{\delta^2+\Gamma^2/4+\Omega^2/2} \\
F_{\text{react}} &= -\frac{\hbar\delta}{4}\frac{\nabla\Omega^2}{\delta^2+\Gamma^2/4+\Omega^2/2}
\end{aligned}
$$

- Reactive force, or dipole force F_{react}
- Dissipative force, or radiation pressure force F_{dissip}

Laser cooling

$$\boldsymbol{F} = \left\langle \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \right\rangle = -\frac{\hbar}{2}\rho_{ge}\left[(\nabla\Omega(\boldsymbol{r})) + \Omega(\boldsymbol{r})(-i)(\nabla\phi(\boldsymbol{r}))\right] e^{-i\phi(\boldsymbol{r})-i\omega t} + c.c.$$
$$= F_{\text{dissip}} + F_{\text{react}}$$

其中

$$F_{\text{dissip}} = \hbar\boldsymbol{k}\frac{\Gamma}{2}\frac{\Omega^2/2}{\delta^2+\Gamma^2/4+\Omega^2/2}$$
$$F_{\text{react}} = -\frac{\hbar\delta}{4}\frac{\nabla\Omega^2}{\delta^2+\Gamma^2/4+\Omega^2/2}$$

- Plane running wave $\rightarrow$ dipole force vanishes
- F_{dissip} saturates to $\hbar\Gamma\boldsymbol{k}/2$

Laser cooling

$$\boldsymbol{F} = \left\langle \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \right\rangle = -\frac{\hbar}{2}\rho_{ge}\left[(\nabla\Omega(\boldsymbol{r})) + \Omega(\boldsymbol{r})(-i)(\nabla\phi(\boldsymbol{r}))\right] e^{-i\phi(\boldsymbol{r})-i\omega t} + c.c.$$
$$= F_{\text{dissip}} + F_{\text{react}}$$

其中

$$F_{\text{dissip}} = \hbar\boldsymbol{k}\frac{\Gamma}{2}\frac{\Omega^2/2}{\delta^2+\Gamma^2/4+\Omega^2/2}$$
$$F_{\text{react}} = -\frac{\hbar\delta}{4}\frac{\nabla\Omega^2}{\delta^2+\Gamma^2/4+\Omega^2/2}$$

- Red detuning $\delta < 0$: dipole force forces the atoms to regions of stronger fields
- Blue detuning $\delta > 0$: dipole force forces the atoms to regions of weak laser intersity

Redistribution of atoms in real space

Doppler cooling

Monochromatic light field

$$\boldsymbol{E}(\boldsymbol{r},t) = \hat{\epsilon}\mathscr{E}\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})$$

- Doppler frequency shift

$$\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{v}_0 t) = \cos[(\omega - \omega_d)t], \quad \omega_d = \boldsymbol{k}\cdot\boldsymbol{v}_0$$

- $\boldsymbol{k}$ and $\boldsymbol{v}_0$ in opposite direction $\rightarrow$ Effective frequency increases
- For moving atom, the effective detuning

$$\delta_d = \omega - \omega_d - \omega_0 = \delta - \omega_d$$

- Dipole force vanish, only radiation-pressure force

$$F_{\mathrm{dissip}} = \frac{1}{2}\hbar\boldsymbol{k}\Gamma\frac{\Omega^2/2}{\delta_d^2 + \Gamma^2/4 + \Omega^2/2}$$

Doppler cooling

Monochromatic light field

$$\boldsymbol{E}(\boldsymbol{r},t)=\hat{\epsilon}\mathscr{E}\cos(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})$$

- Dipole force vanish, only radiation-pressure force

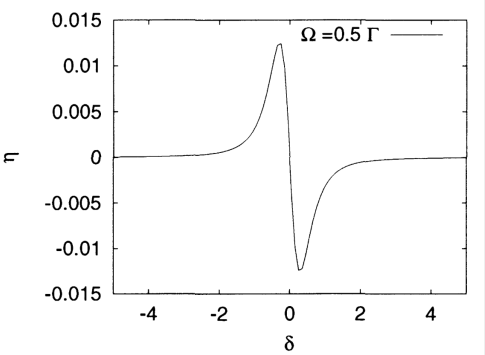
$$F_{\mathrm{dissip}}(v_0)=\frac{1}{2}\hbar\boldsymbol{k}\Gamma\frac{\Omega^2/2}{\delta_d^2+\Gamma^2/4+\Omega^2/2}=F_{\mathrm{dissip}}(v_0)-\eta v_0$$

  where

$$\eta=-\hbar k^2\Gamma\frac{\delta\Omega^2/2}{\left(\delta^2+\Gamma^2/4+\Omega^2/2\right)^2}$$

- A field propagating toward atom $\rightarrow$ Higher frequency

Doppler cooling



For $\delta < 0$ red detuning region:

- Atom moving toward the light field are decelerated
- Atom moving away from the light field are accelerated

Doppler cooling

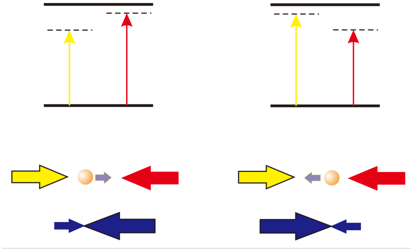
Single running wave field

$$F = \frac{\hbar \boldsymbol{k} \Gamma}{2} \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4 + \Omega^2/2}$$

Two counter propagating fields

$$\begin{aligned} F &= \frac{\hbar \boldsymbol{k} \Gamma}{4} \left[\frac{\Omega^2}{(\delta - \boldsymbol{k} \cdot \boldsymbol{r})^2 + \Gamma^2/4 + \Omega^2/2} - \frac{\Omega^2}{(\delta + \boldsymbol{k} \cdot \boldsymbol{r})^2 + \Gamma^2/4 + \Omega^2/2} \right] \\ &\approx \hbar \Gamma \frac{\delta \boldsymbol{k} (\boldsymbol{k} \cdot \boldsymbol{v}) \Omega^2}{(\delta^2 + \Gamma^2/4 + \Omega^2/2)^2} = -2\eta v_0 \end{aligned}$$

Doppler cooling



- Atoms absorbs a low energy photon, emits a high energy photon
- Energy dissipated through spontaneous emission

Doppler cooling

Each spontaneous emission also result in random momentum diffusion of the atom, or heating

- Diffusive energy change: $\frac{\mathrm{d}E_{\mathrm{diff}}}{\mathrm{d}t} = \frac{\hbar^2 k^2}{2m}\rho_{ee}\Gamma$
- Cooling energy change: $\frac{\mathrm{d}E_{\mathrm{cool}}}{\mathrm{d}t} = \boldsymbol{F} \cdot \boldsymbol{v} = -2\eta v^2$
- At Doppler limit $\delta = -\Gamma/2$

$$\frac{\mathrm{d}E_{\mathrm{cool}}}{\mathrm{d}t} + \frac{\mathrm{d}E_{\mathrm{diff}}}{\mathrm{d}t} = 0 \Longrightarrow k_B T_D \sim \frac{\hbar\Gamma}{16}$$

- More sophiscated theory gives $k_B T_D \sim \frac{\hbar\Gamma}{2}$

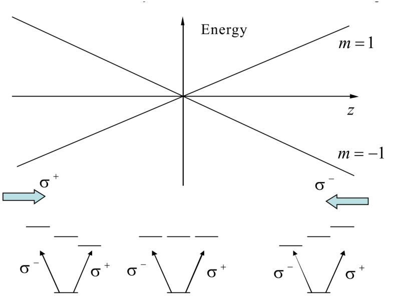
Doppler cooling

Limit of Doppler cooling

Atom	T_D (μK)	T_{recoil} (μK)
^{1}H	2389	1285
$^{7}\mathrm{Li}$	142.11	0.06
$^{23}\mathrm{Na}$	240.18	2.40
$^{85}\mathrm{Rb}$	143.41	0.37
$^{133}\mathrm{Cs}$	124.39	0.20

Magneto-optical trap (MOT)

Combine Doppler cooling and spatially varying Zeeman effect to cool and trap atoms



For one-dimensional MOT, the magnetic field $\boldsymbol{B} = Az$. Detuning depends on both velocity and position

$$\delta_{\pm} = \delta \mp \boldsymbol{k} \cdot \boldsymbol{v} \mp \beta z$$

- Atoms at $z < 0$ are closer to resonance of σ^{+} beam
- Atoms at $z > 0$ are closer to resonance of σ^{-} beam

Magneto-optical trap

The total frictional force

$$\boldsymbol{F} \approx 2\hbar\Omega^2\Gamma\frac{\delta \boldsymbol{k}(\boldsymbol{k}\cdot\boldsymbol{v}+\beta z)}{(\delta^2+\Gamma^2/4)^2}$$

- Classical damped harmonic oscillator
- Limitations: Radiation, many-body collision

Exmaple: Rb MOT, $n \sim 10^{11}$ cm^{-3}, $T \sim 100\,\mu$K

Sisyphus cooling

- 1988, Philips found the limit temperature of Na $40\,\mu$K, much lower than $240\,\mu$K predicted by Doppler cooling

What is missing?

- Two counter propagating fields add up inhomogeneous
- Realistic atoms have more complicated level structures

西西弗斯，是希腊神话中一位被惩罚的人。他受罚的方式是：必须将一块巨石推上山顶，而每次到达山顶后巨石又滚回山下，如此永无止境地重复下去。

Sisyphus cooling

Two counterpropagating light, so-called lin-⊥-lin geometry

$$\begin{aligned} \boldsymbol{E}(z,t) &= \frac{1}{2}\left[\boldsymbol{\epsilon}_x \mathscr{E}_0 e^{i(kz-\omega t)} - i\boldsymbol{\epsilon}_y \mathscr{E}_0 e^{-i(kz+\omega t)} + c.c.\right] \\ &= \frac{1}{2}\left[\boldsymbol{\epsilon}(z)\mathscr{E} e^{-i\omega t} + c.c.\right] \end{aligned}$$

The right/left-circular polarization

$$\boldsymbol{\epsilon}_+ = \frac{\sqrt{2}}{2}(\varepsilon_x - i\varepsilon_y), \quad \boldsymbol{\epsilon}_- = -\frac{\sqrt{2}}{2}(\varepsilon_x + i\varepsilon_y)$$

and

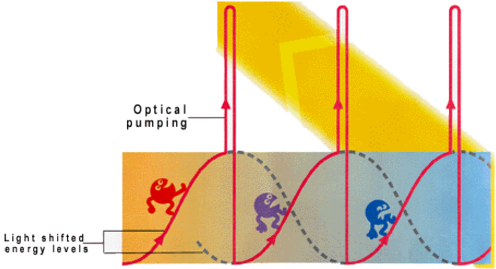
$$\boldsymbol{\epsilon}(z) = \boldsymbol{\epsilon}_- \cos(kz) - i\boldsymbol{\epsilon}_+ \sin(kz)$$

- Spatially varying polarization
- Spin-dependent lattice potentials

Sisyphus cooling

Lin-⊥-Lin geometry

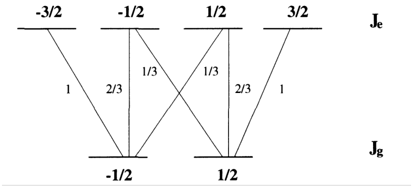
$$\boldsymbol{E} = \frac{\sqrt{2}\mathscr{E}}{2} e^{-i\omega t} [\boldsymbol{\epsilon}_- \cos(kz) - i\boldsymbol{\epsilon}_+ \sin(kz)]$$



- Atoms hop to the other ground state near maxima of potential via optical pumping

Sisyphus cooling

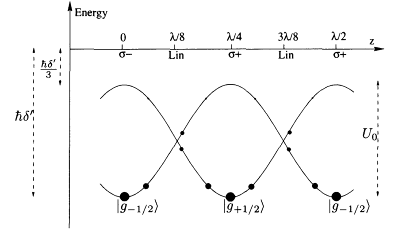
Selective rule and CG coeffcients



For the $J_g = 1/2 \leftrightarrow J_e = 3/2$:

$$H = -\frac{i}{2}\hbar\Omega\sin(kz)\left(\left|e_{3/2}\middle\rangle\middle\langle g_{1/2}\right| + \frac{1}{\sqrt{3}}\left|e_{1/2}\middle\rangle\middle\langle g_{-1/2}\right|\right)e^{-i\omega t}$$
$$+\frac{1}{2}\hbar\Omega\cos(kz)\left(\left|e_{-3/2}\middle\rangle\middle\langle g_{-1/2}\right| + \frac{1}{\sqrt{3}}\left|e_{-1/2}\middle\rangle\middle\langle g_{1/2}\right|\right)e^{-i\omega t} + \mathrm{H.c.}$$

Sisyphus cooling



AC stark shift of the ground states via large detuning approximation:

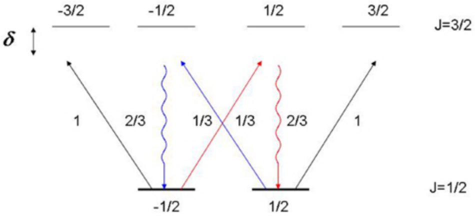
$$U = \hbar\delta s_0 \left[1 - \frac{2}{3}\cos^2(kz)\right] |g_{1/2}\rangle\langle g_{1/2}| + \hbar\delta s_0 \left[1 - \frac{2}{3}\sin^2(kz)\right] |g_{-1/2}\rangle\langle g_{-1/2}|$$
$$= U_+(z) |g_{1/2}\rangle\langle g_{1/2}| + U_-(z) |g_{-1/2}\rangle\langle g_{-1/2}|$$

where

$$s_0 = \frac{1}{2}\frac{\Omega^2}{\delta^2 + \Gamma^2/4}$$

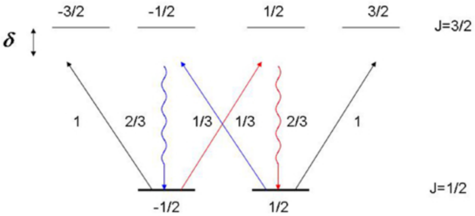
- Minima of $U_-(z)$ at $z = n\lambda/2$, where the polarization of the laser field is σ^-
- Minima of $U_+(z)$ at $z = \lambda/4 + n\lambda/2$, where the polarization of the laser field is σ^+

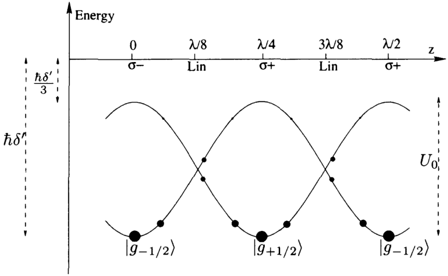
Sisyphus cooling



- Atoms move up the potential "hill" → decelerated
- At maxima of the "hill" → transition to excited state → stimulated back to ground state

Sisyphus cooling





Sisyphus cooling

Rate equations from master equation

$$\frac{\mathrm{d}P_{1/2}}{\mathrm{d}t} = -\frac{2\Gamma s_0}{9}\cos^2(kz)P_{1/2} + \frac{2\Gamma s_0}{9}\sin^2(kz)P_{-1/2}$$
$$\frac{\mathrm{d}P_{-1/2}}{\mathrm{d}t} = -\frac{\mathrm{d}P_{1/2}}{\mathrm{d}t}$$

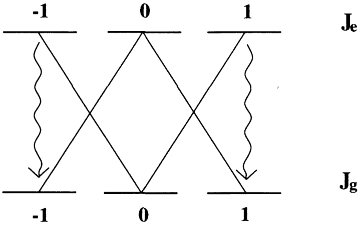
- Red-detuning laser $\delta < 0$
- Loss rate from $|g_{1/2}\rangle$ maximum near the maximum of polarization gradient potential
- Rate equation symmetric $g_{1/2}$ and $g_{-1/2}$
- Sisyphus cooling limit for $^{23}\mathrm{Na}$: $T_s \sim 2.4\,\mu$K

Subrecoil cooling

- Temperature in previous cooling schemes limited by spontaneous recoil energy
- Avoid spontaneous emission
- Drive atoms to a “dark state”

Subrecoil cooling

Consider $J_g = 1 \leftrightarrow J_e = 1$:



The combined laser fields

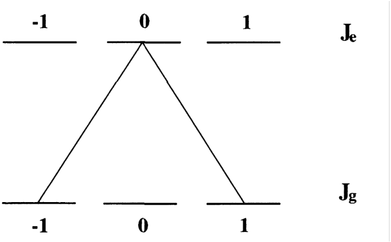
$$\boldsymbol{E}(z,t) = \frac{1}{2}\mathscr{E}\left(\boldsymbol{\epsilon}_+ e^{ikz} + \boldsymbol{\epsilon}_- e^{-ikz}\right)$$

- V-type and Λ-type three-level system
- Population can go from V to Λ system, but not the other

Ignore the levels associated with V-system

Subrecoil cooling

Consider $J_g = 1 \leftrightarrow J_e = 1$:



The combined laser fields

$$\boldsymbol{E}(z,t) = \frac{1}{2}\mathscr{E}\left(\boldsymbol{\epsilon}_+ e^{ikz} + \boldsymbol{\epsilon}_- e^{-ikz}\right)$$

- V-type and Λ-type three-level system
- Population can go from V to Λ system, but not the other

Ignore the levels associated with V-system

Subrecoil cooling

The coupling of laser field and atom

$$H = \frac{\hbar\Omega}{2\sqrt{2}} \left(|e_0\rangle\langle g_1|\, e^{-ikz} - |e_0\rangle\langle g_{-1}|\, e^{ikz} \right) e^{-i\omega t} + \mathrm{H.\,c.}$$

- Note the sign different from Chapter 5

Subrecoil cooling

The coupling of laser field and atom

$$H = \frac{\hbar\Omega}{2\sqrt{2}} \int \mathrm{d}p \left(|e_0, p\rangle\langle g_1, p + \hbar k| - |e_0, p\rangle\langle g_{-1}, p - \hbar k| \right) e^{-i\omega t} + \mathrm{H.\,c.}$$

- Note the sign different from Chapter 5
- Need to involve momentum
- The translation operator $e^{ikz} = \int \mathrm{d}p \, |p\rangle\langle p - \hbar k|$

Subrecoil cooling

Due to conservation of momentum

- Closed manifolds

$$\mathcal{M}(p) = \{|e_0, p\rangle, |g_1, p+\hbar k\rangle, |g_{-1}, p-\hbar k\rangle\}$$

- In this basis

$$H = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & 0 & \Omega^* \\ 0 & 0 & -\Omega^* \\ \Omega & \Omega & -\delta \end{pmatrix}$$

- New basis

$$|\psi_c(p)\rangle = \frac{1}{\sqrt{2}} \left(|g_1, p+\hbar k\rangle - |g_{-1}, p-\hbar k\rangle\right)$$
$$|\psi_{nc}(p)\rangle = \frac{1}{\sqrt{2}} \left(|g_1, p+\hbar k\rangle + |g_{-1}, p-\hbar k\rangle\right)$$
$$|\psi_e(p)\rangle = |e_0, p\rangle$$

Satisfy

$$H|\psi_c\rangle = \frac{\hbar}{2\sqrt{2}}\Omega|\psi_e\rangle, \quad H|\psi_e\rangle = \frac{\hbar\Omega}{2\sqrt{2}}|\psi_c\rangle, \quad H|\psi_{nc}\rangle = 0$$

Subrecoil cooling

- Note that $|\psi_{nc}(p)\rangle$ is not exactly a dark state
- Take kinetic energy as perturbation

$$\langle\psi_c(p)|\frac{p^2}{2m}|\psi_{nc}(p)\rangle = \frac{\hbar k p}{M}$$

- Atoms can escape from the state $|\psi_{nc}(p)\rangle$, except for $p = 0$
- $\psi_{nc}(0)$ is a perfect trap, a dark state uncoupled to any other state of the system

Subrecoil cooling

- Linewidth of upper state $|\psi_e(p)\rangle$: Γ
- Linewidth of $\psi_c(p)$: $\Gamma' = \frac{\Omega^2}{\Gamma}$
- Linewidth of $\psi_{nc}(p)$:

$$\Gamma'' = 4\left(\frac{kp}{M}\right)^2 \frac{\Gamma}{\Omega^2}$$

when $kp/M \ll \Gamma'$

Evaporative cooling

Limitations

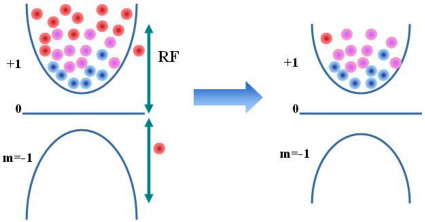
- Doppler/Sisyphus cooling in MOT are limited by heating due to spontaneous emission
- MOT also exerts density limits due to trap losses etc.
- Subrecoil coolings have not reached the phase space densities required for a condensate

Evaporative cooling

- Remove atoms with highest energies from the trap
- Temperature lowered as atoms rethermalized via elastic collison

Evaporative cooling

Evaporative cooling in a magnetic trap



- Need traps with coservative forces: magnetic trap, optical dipole trap
- Lifetime of gas much longer than collision rate
- Ineffective for identical fermions

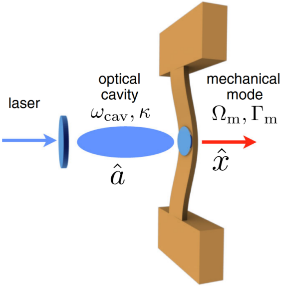
Evaporative cooling

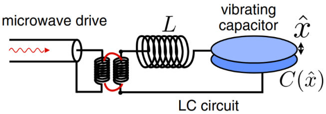
Group	Atom	Eva. Cooling	N (10^6)	n (10^{12} cm^{-3})	T (μK)	ρ (10^{-6})
Rice	^{7}Li	Before	200	0.07	200	7
		After	0.1	1.4	0.4	
MIT	^{23}Na	Before	1000	0.1	200	2
		After	0.7	150	2	
JILA	^{87}Rb	Before	4	0.04	90	0.3
		After	0.02	3	0.17	

Microscopic$\leftrightarrow$ Macroscopic

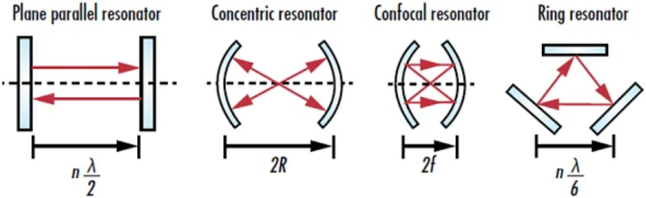
Mesoscopic/Macroscopic quantum effect?

Cavity Optomechanics



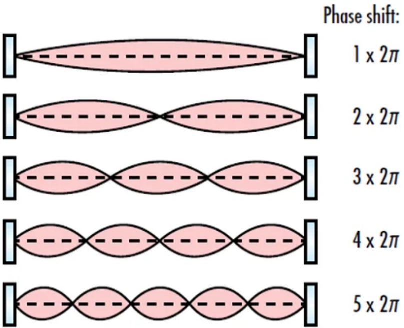


Cavity + Mechanical resonator



Different cavities

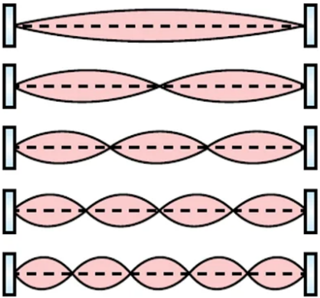
Cavity Optomechanics



Fabry-Pérot resonator:

$$m\lambda = 2L \Longrightarrow \omega = \frac{2\pi c}{\lambda} = m\frac{\pi c}{L}$$

Cavity Optomechanics



Fabry-Pérot resonator:

$$m\lambda = 2L \Longrightarrow \omega = \frac{2\pi c}{\lambda} = m\frac{\pi c}{L}$$

- Optomechanics: one mirror of the cavity is movable

Cavity Optomechanics

- Cavity decay rate κ: Mirror transparencies, and internal absorption or scattering out of the cavity
- Optical finesse (精细度): average number of round-trips before a photon leaves cavity

$$\mathcal{F} = \frac{\Delta\omega}{\kappa}$$

- Quality factor (品质因子) of the optical resonator

$$Q = 2\pi \frac{\text{Energy of cavity}}{\text{Decay of energy per round}} = \omega_{\text{cav}}\tau$$

where $\tau = 1/\kappa$ is the photon lifetime

Cavity Optomechanics

Mechanical resonators

$$m_{\rm eff}\frac{{\rm d}^2x(t)}{{\rm d}t^2}+m_{\rm eff}\Gamma_m\frac{{\rm d}x(t)}{{\rm d}t}+m_{\rm eff}\Omega_m^2x(t)=F_{\rm ex}(t)$$

Phonon creation and annihilation operators

$$\hat{x}=x_{\rm ZPF}(\hat{b}+\hat{b}^\dagger),\quad \hat{p}=-im_{\rm eff}\Omega_m x_{\rm ZPF}(\hat{b}-\hat{b}^\dagger)$$

where the zero-point fluctuation

$$x_{\rm ZPF}=\sqrt{\frac{\hbar}{2m_{\rm eff}\Omega_m}}$$

- Define the susceptibility χ_m via Fourier transform: $x(\omega) = \chi_m(\omega) F_{\text{ex}}(\omega)$

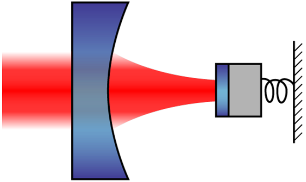
$$\chi_m(\omega) = \frac{1}{m_{\text{eff}}(\Omega_m^2 - \omega^2) - i m_{\text{eff}} \Gamma_m \omega}$$

- Low-frequency response

$$\chi_m(0) = \frac{1}{m_{\text{eff}} \Omega_m^2} = \frac{1}{k},$$

where k is the spring constant

Cavity Optomechanics



- Momentum transfer of single photon $|\Delta p| = 2h/\lambda = 2\hbar k$
- Radiation-pressure force

$$\langle F \rangle = \frac{\Delta p}{\tau_c} \langle n \rangle = \hbar \frac{\omega_{\text{cav}}}{L} \left\langle \hat{a}^\dagger \hat{a} \right\rangle$$

where $\tau_c = 2L/c$ denotes the cavity round-trip time

- $G = \omega_{\text{cav}}/L$ describes the change of cavity resonance frequency with position

Cavity Optomechanics

$$H_0 = \hbar\omega_{\text{cav}}\hat{a}^\dagger\hat{a} + \hbar\Omega_m\hat{b}^\dagger\hat{b}$$

- The cavity resonance frequency is modulated by the mechanical amplitude

$$\omega_{\text{cav}}(x) \approx \omega_{\text{cav}} + x\frac{\partial\omega_{\text{cav}}}{\partial x} + \dots$$

- Optical frequency shift per displacement

$$G = -\frac{\partial\omega_{\text{cav}}}{\partial x}$$

- Expanding to leading order

$$\hbar\omega_{\text{cav}}(x)\hat{a}^\dagger\hat{a} \approx \hbar(\omega_{\text{cav}} - G\hat{x})\hat{a}^\dagger\hat{a}$$

- Interaction part of the Hamiltonian

$$H_{\text{int}} = -\hbar g_0\hat{a}^\dagger\hat{a}(\hat{b} + \hat{b}^\dagger), \quad g_0 = Gx_{\text{ZPF}}$$

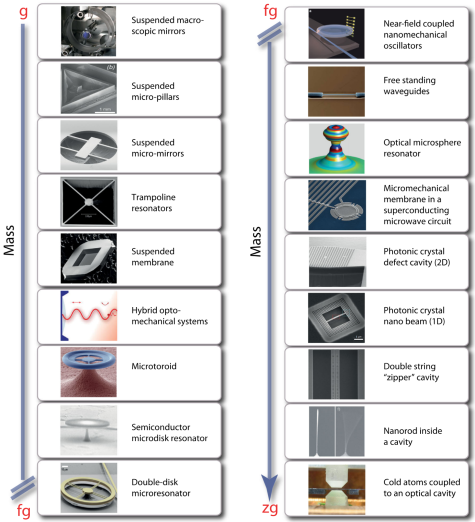
- Nonlinear process: three-wave mixing

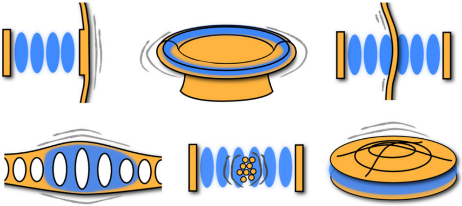
Cavity Optomechanics

$$\omega_{\text{cav}} \sim 10^{14}\,\text{Hz}, \quad \Omega \sim 10^{9}\,\text{Hz}$$

g_0 smaller

Cavity Optomechanics





Radiation modes and vibrational degree of freedom

Cavity Optomechanics

TABLE II. Experimental parameters for a representative sampling of published cavity-optomechanics experiments.

Reference	$\Omega_m/2\pi$ (Hz)	m (kg)	$\Gamma_m/2\pi$ (Hz)	Qf (Hz)	$\kappa/2\pi$ (Hz)	κ/Ω_m	$g_0/2\pi$ (Hz)
Murch *et al.* (2008)	4.2×10^4	1×10^{-22}	1×10^3	1.7×10^6	6.6×10^5	15.7	6×10^5
Chan *et al.* (2011)	3.9×10^9	3.1×10^{-16}	3.9×10^4	3.9×10^{14}	5×10^8	0.13	9×10^5
Teufel, Donner *et al.* (2011)	1.1×10^7	4.8×10^{-14}	32	3.5×10^{12}	2×10^5	0.02	2×10^2
Verhagen *et al.* (2012)	7.8×10^7	1.9×10^{-12}	3.4×10^3	1.8×10^{12}	7.1×10^6	0.09	3.4×10^3
Thompson *et al.* (2008)	1.3×10^5	4×10^{-11}	0.12	1.5×10^{11}	5×10^5	3.7	5×10^1
Kleckner *et al.* (2011)	9.7×10^3	1.1×10^{-10}	1.3×10^{-2}	9×10^9	4.7×10^5	55	2.2×10^1
Gröblacher, Hammerer *et al.* (2009)	9.5×10^5	1.4×10^{-10}	1.4×10^2	6.3×10^9	2×10^5	0.22	3.9
Arcizet *et al.* (2006a)	8.14×10^5	1.9×10^{-7}	81	8.1×10^9	1×10^6	1.3	1.2
Cuthbertson *et al.* (1996)	10^3	1.85	2.5×10^{-6}	4.1×10^{10}	275	0.9	1.2×10^{-3}

Cavity Optomechanics

Hamiltonian

$$H = \hbar\omega_{\mathrm{cav}}\hat{a}^\dagger\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} + \hbar g_0\hat{a}^\dagger\hat{a}(\hat{b}+\hat{b}^\dagger)$$

Quantum Langevin equation

$$\dot{\hat{a}} = \frac{1}{i\hbar}\left[\hat{a},\hat{H}\right] - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\hat{a}_{\mathrm{in}}$$
$$\dot{\hat{b}} = \frac{1}{i\hbar}\left[\hat{b},\hat{H}\right] - \frac{\Gamma}{2}\hat{b}$$

In the stable state, let $\dot{\hat{a}} = 0$ and $\dot{\hat{b}} = 0$ to find mean field solution $\alpha = \langle\hat{a}\rangle$

$$\dot{\alpha} = -i\left(\Delta + g_0\left\langle\hat{b}+\hat{b}^\dagger\right\rangle\right)\alpha - \frac{\kappa}{2}\alpha + \sqrt{\kappa}\alpha_{\mathrm{in}} = 0$$
$$\left\langle\dot{\hat{b}}\right\rangle = -i\Omega\left\langle\hat{b}\right\rangle - ig_0|\alpha|^2 - \frac{\Gamma}{2}\left\langle\hat{b}\right\rangle = 0$$

One can find

$$\left\langle \hat{b} \right\rangle = -\frac{g_0 |\alpha|^2}{\Omega - i\Gamma/2}$$

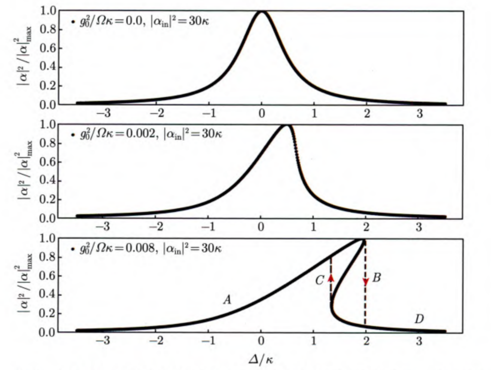
and

$$\left\langle \hat{b} + \hat{b}^\dagger \right\rangle = -\frac{2\Omega g_0 |\alpha|^2}{\Omega^2 + \Gamma^2/4}$$

The photon number

$$|\alpha|^2 \approx \frac{\kappa |\alpha_{\text{in}}|^2}{\kappa^2/4 + (\Delta - 2g_0^2 |\alpha|^2/\Omega)^2}$$

Cavity Optomechanics



Multistable and first-order phase transition

大湾区大学

Cavity Optomechanics

Quantum fluctuation

$$\hat{a} \to \alpha + \delta\hat{a}, \quad \hat{b} \to \beta + \delta\hat{b}$$

Effective Hamiltonian

$$H = \hbar\left(\Delta - \frac{2g_0^2|\alpha|^2}{\Omega}\right)\delta\hat{a}^\dagger\delta\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b}$$
$$+ \hbar g_0\left[\alpha(\delta\hat{a} + \delta\hat{a}^\dagger) + \delta\hat{a}^\dagger\delta\hat{a}\right](\hat{b} + \hat{b}^\dagger)$$

- Effective detuning
- Linearized approximation

$$H = \hbar\Delta\delta\hat{a}^\dagger\delta\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} + \hbar g_0\alpha(\delta\hat{a} + \delta\hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$

- Rotating-wave approximation ($\Delta > 0$, weak coupling)

$$H = \hbar\Delta\delta\hat{a}^\dagger\delta\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} + \hbar g_0\alpha\left(\delta\hat{a}\hat{b}^\dagger + \delta\hat{a}^\dagger\hat{b}\right)$$

Cavity Optomechanics

Quantum fluctuation

$$\hat{a} \to \alpha + \delta\hat{a}, \quad \hat{b} \to \beta + \delta\hat{b}$$

Effective Hamiltonian

$$H = \hbar\left(\Delta - \frac{2g_0^2|\alpha|^2}{\Omega}\right)\delta\hat{a}^\dagger\delta\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} + \hbar g_0\left[\alpha(\delta\hat{a} + \delta\hat{a}^\dagger) + \delta\hat{a}^\dagger\delta\hat{a}\right](\hat{b} + \hat{b}^\dagger)$$

- Effective detuning
- Linearized approximation

$$H = \hbar\Delta\delta\hat{a}^\dagger\delta\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} + \hbar g_0\alpha(\delta\hat{a} + \delta\hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$

- Rotating-wave approximation ($\Delta < 0$, weak coupling)

$$H = \hbar\Delta\delta\hat{a}^\dagger\delta\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} + \hbar g_0\alpha\left(\delta\hat{a}^\dagger\hat{b}^\dagger + \delta\hat{a}\hat{b}\right)$$

Cavity Optomechanics

Linearized version of the classical equations of motion

$$\delta\dot{\alpha} = \left(-i\Delta - \frac{\kappa}{2}\right)\delta\alpha - iG\bar{\alpha}x,$$
$$m_{\text{eff}}\ddot{x} = -m_{\text{eff}}\Omega_m^2 x - m_{\text{eff}}\Gamma_m\dot{x} - \hbar G(\bar{\alpha}^*\delta\alpha + \bar{\alpha}\delta\alpha^*)$$

The modified susceptibility

$$\chi_{\text{m,eff}}^{-1} = \chi_m^{-1}(\omega) + \Sigma(\omega),$$
$$\Sigma(\omega) = 2m_{\text{eff}}\Omega_m g^2\left(\frac{1}{\Delta+\omega+i\kappa/2} + \frac{1}{\Delta-\omega-i\kappa/2}\right)$$

Effective result: frequency shift and optomechanical damping

$$\delta\Omega_m(\omega) = g^2\frac{\Omega_m}{\omega}\left[\frac{\Delta+\omega}{(\Delta+\omega)^2+\kappa^4/4} + \frac{\Delta-\omega}{(\Delta-\omega)^2+\kappa^2/4}\right]$$
$$\Gamma_{\text{opt}}(\omega) = g^2\frac{\Omega_m}{\omega}\left[\frac{\kappa}{(\Delta+\omega)^2+\kappa^2/4} - \frac{\kappa}{(\Delta-\omega)^2+\kappa^2/4}\right]$$

Cavity Optomechanics

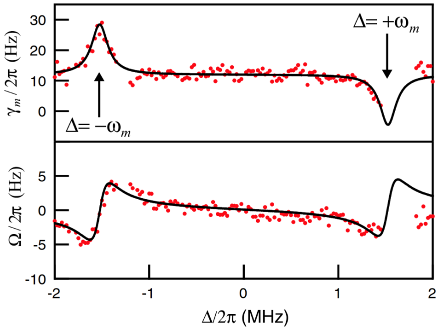
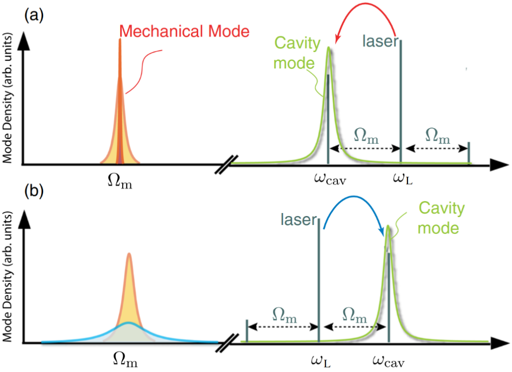


FIG. 17 (color online). Optomechanical cooling rate and frequency shift in the resolved-sideband regime, shown here for $\kappa/\Omega_m \ll 1$. Adapted from Teufel *et al.*, 2008.

- Cooling region: $\Delta = -\omega_m$, maximum of γ_m, and zero frequency shift
- Amplication region: $\Delta = \omega_m$, minimum of γ_m, and zero frequency shift

Cavity Optomechanics

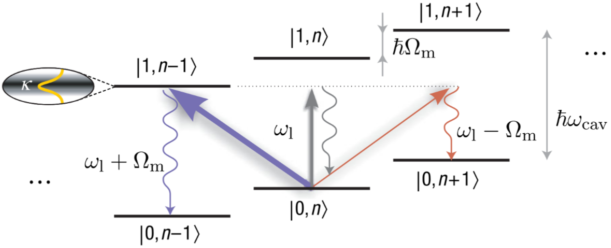
Scattering picture



- Stokes process
- Anti-Stokes process

Cavity Optomechanics

For the case $\omega_c - \omega_l = \Omega_m$



Principle of cavity-optomechanical cooling and corresponding transition diagram

Cavity Optomechanics

- Photons enter cavity resonance by obsorbing a phonon from oscillator
- Anti-Stokes processes

$$\Gamma_{n\to n-1} = nA^-$$

- Stokes processes happen at a smaller rate

$$\Gamma_{n\to n+1} = (n+1)A^+$$

- Full optomechnical damping rate $\Gamma_{\mathrm{opt}} = A^- - A^+$

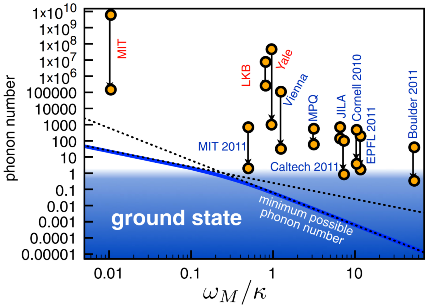
$$\dot{n} = (n+1)(A^+ + A^+_{th}) - n(A^- + A^-_{th})$$

- Steady-state final phonon number

$$n_f = \frac{A^+ + n_{th}\Gamma_m}{\Gamma_{opt} + \Gamma_m}$$

The transition rates can be calculated by the quantum noise spectrum

$$A^{\pm} = \frac{x_{\mathrm{ZPF}}^2}{\hbar^2} S_{FF}(\omega = \mp\Omega_m) = g_0^2 S_{NN(\omega=\mp\Omega_m)}$$



Experimental results for optomechanical laser cooling

Summary

Applications of quantum optics

- Ultracold atom physics
- Laser cooling
- Quantum steering of single atom and atomic ensemble
- Coupling between microscopic degree of freedom (optical caivyt mode) and macroscopic degree of freedom (vibrating mirror)
- Macroscopic quantum effect