Quantum optics (2026), **1**:1-5 1–1

Homework

Problems of Chapter 1

1. 在边长为 L 的立方体腔内，辐射场在库仑规范 $\nabla \cdot \boldsymbol{A} = 0$ 下满足波动方程

$$\nabla^2 \hat{\boldsymbol{A}} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = 0,$$

考虑边界条件，电场的切向分量必须为零，请推导出 $\boldsymbol{A}$ 的三个分量满足以下关系：

$$\begin{aligned} A_x(\boldsymbol{r}, t) &= A_x(t)\cos(k_x x)\sin(k_y y)\sin(k_z z) \\ A_y(\boldsymbol{r}, t) &= A_y(t)\sin(k_x x)\cos(k_y y)\sin(k_z z) \\ A_z(\boldsymbol{r}, t) &= A_z(t)\sin(k_x x)\sin(k_y y)\cos(k_z z). \end{aligned}$$

其中 $k_i = 2\pi n_i / L$, $n_i = 0, \pm 1, \pm 2, \ldots$, $i = x, y, z$.

2. $\hat{A}$ 和 $\hat{B}$ 为两个力学量算符，证明

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + \left[\hat{A}, \hat{B}\right] + \frac{1}{2!}\left[\hat{A}, [\hat{A}, \hat{B}]\right] + \ldots$$

3. $\hat{a}$ 和 $\hat{a}^\dagger$ 分别是玻色子的湮灭算符和产生算符，满足 $\hat{\hat{a}}\hat{a}^\dagger = 1$，证明以下关系式

   (a) $\left[\hat{a}, f(\hat{a}, \hat{a}^\dagger)\right] = \frac{\partial f}{\partial \hat{a}^\dagger}$

   (b) $\left[\hat{a}^\dagger, f(\hat{a}, \hat{a}^\dagger)\right] = -\frac{\partial f}{\partial \hat{a}}$

   (c) $e^{-\alpha \hat{a}^\dagger \hat{a}} f(\hat{a}, \hat{a}^\dagger) e^{\alpha \hat{a}^\dagger \hat{a}} = f(\hat{a}e^{\alpha}, \hat{a}^\dagger e^{-\alpha})$, 其中 α 是个常数

4. 证明以下关系：

$$\begin{aligned} \left[\hat{a}, e^{-\alpha \hat{a}^\dagger \hat{a}}\right] &= (e^{-\alpha} - 1)e^{-\alpha \hat{a}^\dagger \hat{a}}\hat{a} \\ \left[\hat{a}^\dagger, e^{-\alpha \hat{a}^\dagger \hat{a}}\right] &= (e^{\alpha} - 1)e^{-\alpha \hat{a}^\dagger \hat{a}}\hat{a}^\dagger \end{aligned}$$

5. 针对以下问题说说你的理解：(1) 为什么在 Fock 态中，电磁场的期望是 0? (2) 为什么在基态 $|0\rangle$ 涨落不是 0?